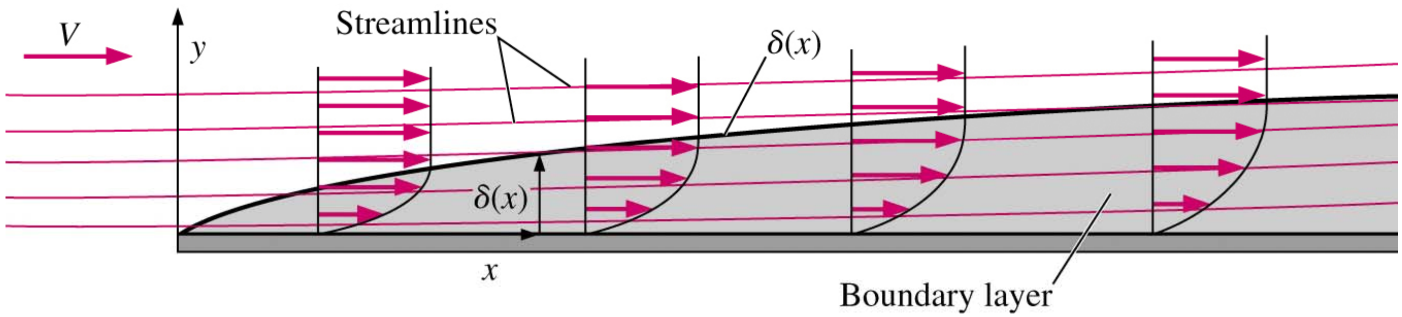
Boundary Layer

<u>Boundary Layer</u> - layer of fluid near the surface that has undergone a change in velocity because of the shear stress at the surface.



At the wall the fluid particles are attached to the wall (no slip b.c.)

$$u = 0$$

Assume the thickness of boundary layer is where

$$u = 0.99U$$

Boundary Layer

Note when velocity gradient is small, shear stress is small.

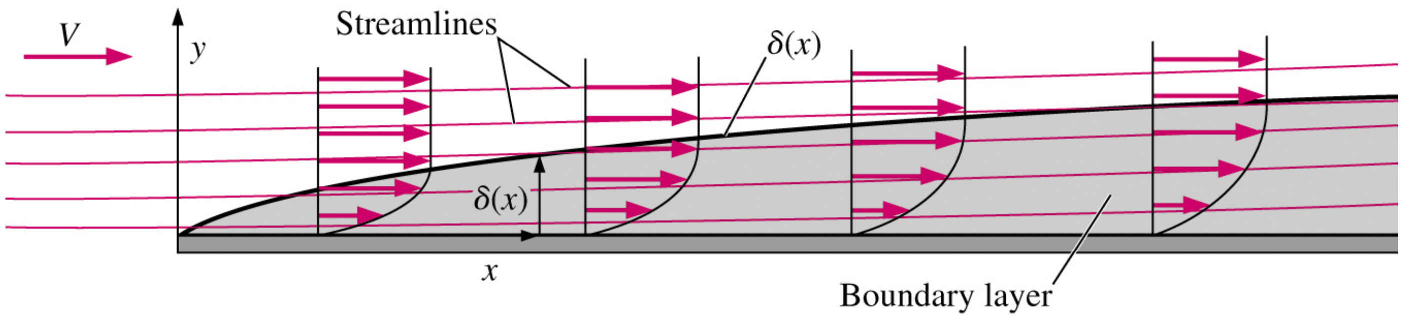
$$\tau = \mu \frac{\partial u}{\partial y}$$

At the wall

$$\tau = \tau_o$$

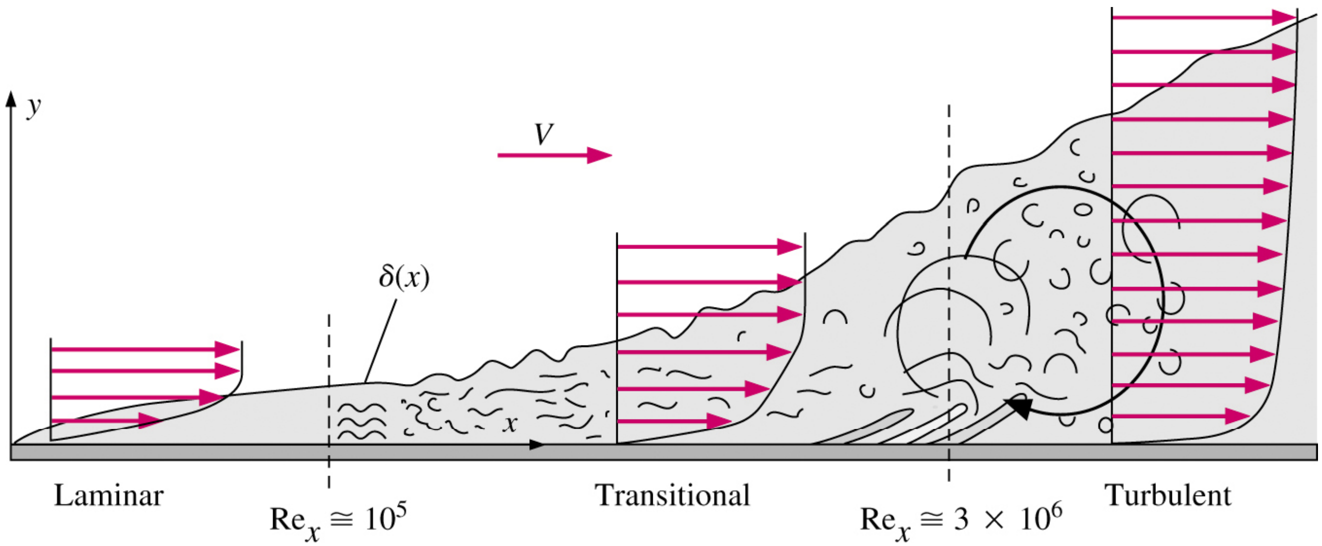
At the boundary layer thickness, y = δ

$$\tau = 0$$



Boundary Layer Transition

- At some distance downstream on the flat plate, disturbances grow and laminar boundary layer transitions to turbulent boundary layer.



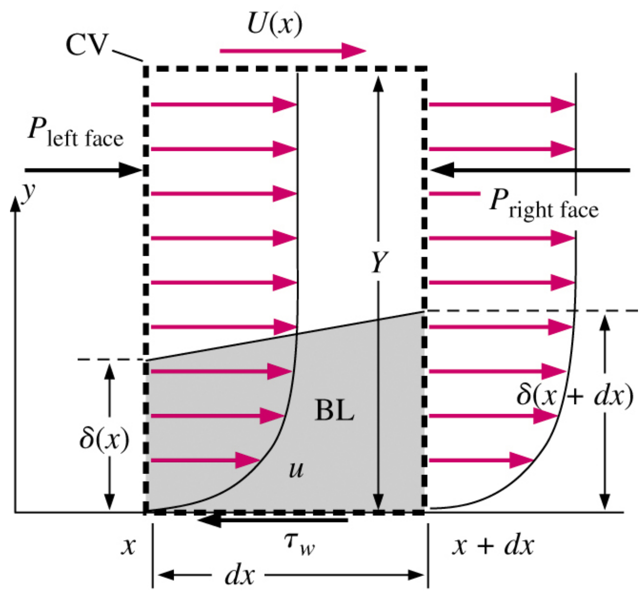
$$\mathrm{Re}_{x,critical} \cong 1\times10^5 \text{ to } 5\times10^5 \qquad \text{(transition begins)}$$

$$\mathrm{Re}_{x,transitional} \cong 3\times10^6 \qquad \text{(fully turbulent)}$$

Von Karman Integral

- Combining the integral form of the continuity and momentum equations results in the integral.

$$\tau_w = \rho U^2 \frac{d}{dx}\left[\int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy\right]$$



- Valid for laminar or turbulent flow.
- Requires knowing the velocity profile in the boundary layer.

Von Karman Integral

- Assuming a third order polynomial

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

- Result is the boundary layer thickness.

$$\delta = 4.65 \frac{x}{\sqrt{\mathrm{Re}_x}}$$

Von Karman Integral

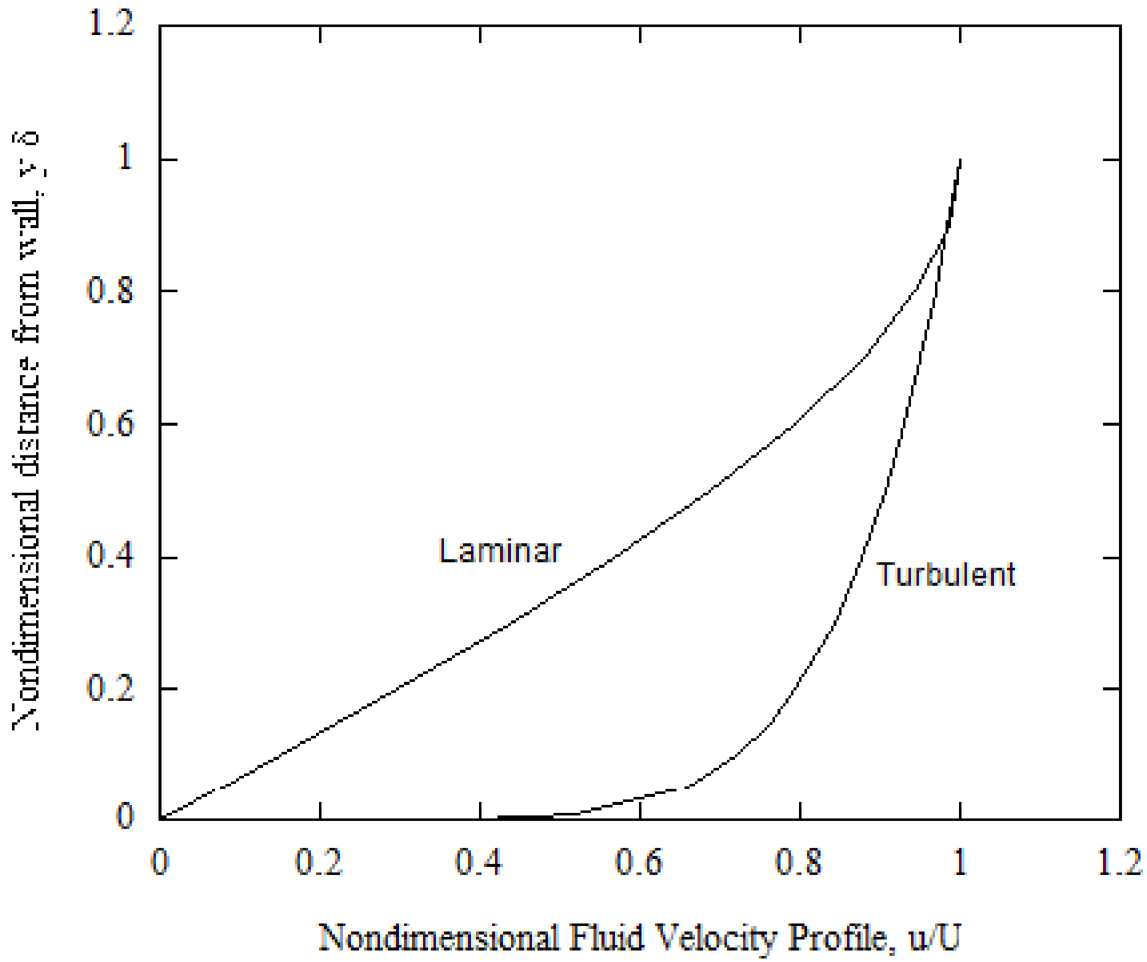
- Assuming power law form for turbulent flow.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/n} \qquad n = \begin{cases} 7 & \mathrm{Re_x} < 10^7 \\ 8 & 10^7 < \mathrm{Re_x} < 10^8 \\ 9 & 10^8 < \mathrm{Re_x} < 10^9 \end{cases}$$

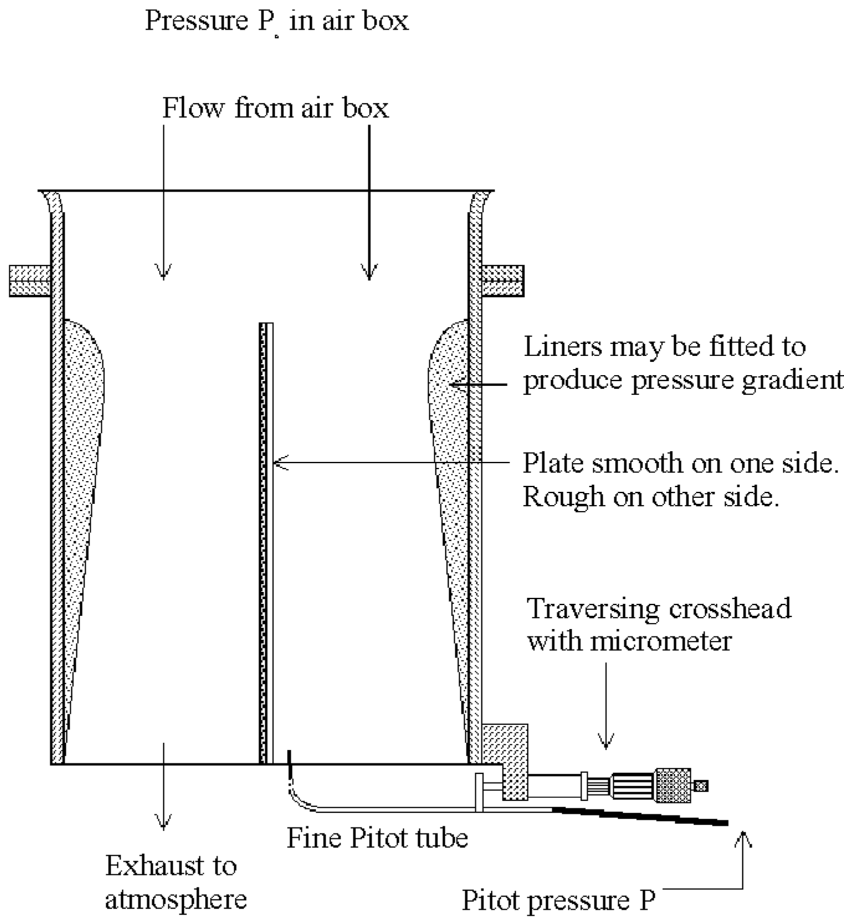
- Result is the boundary layer thickness (n = 7).

$$\delta = 0.38 \frac{x}{\mathrm{Re}_x^{1/5}}$$

Laminar vs. Turbulent Velocity Profile



Experiment Apparatus

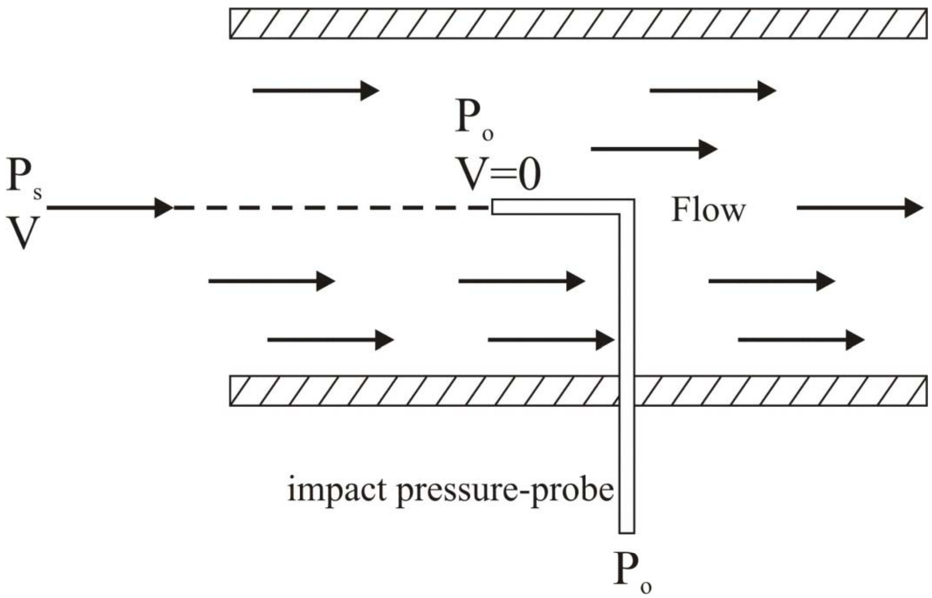


- Measure air temperature (thermometer)
- Measure air velocity (pitot tube)
- Pitot tube position from plate is controlled by micrometer. Its measurement is relative.

- Blower forces air through duct.
- User controls air flow rate

Pressure Probe for Measuring Fluid Velocity

- A small diameter tube is placed with its opening perpendicular to the fluid velocity.
- The fluid at the opening of the tube has zero velocity and thus represents the stagnation pressure or total pressure.



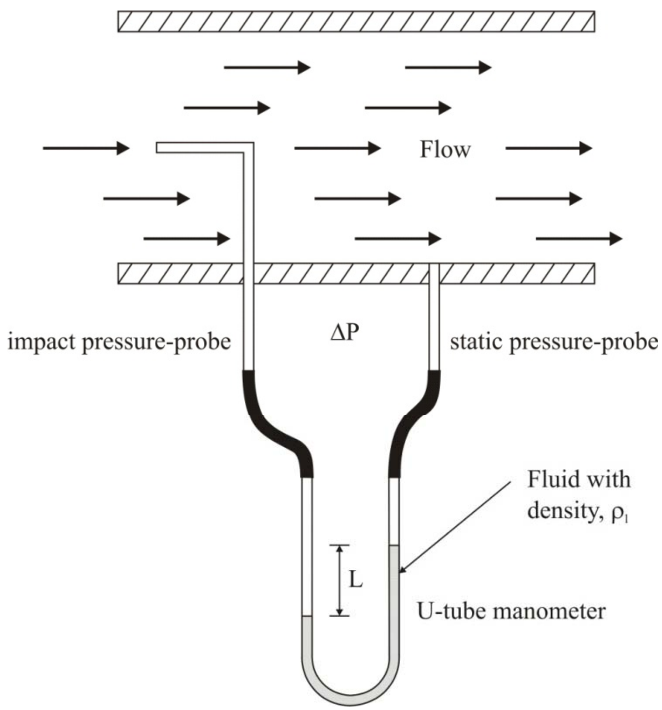
$$P_o = P_s + \frac{1}{2 g_c} \rho V^2$$

$$V = \sqrt{\frac{2 g_c (P_o - P_s)}{\rho}}$$

Pressure Probe for Measuring Fluid Velocity

- A manometer is used to measure the difference between stagnation pressure and static pressure.

- Based on the hydrostatic equation, a liquid column of fluid can be converted to a pressure differential.



$$\Delta P = \frac{\rho_l g}{g_c} L$$