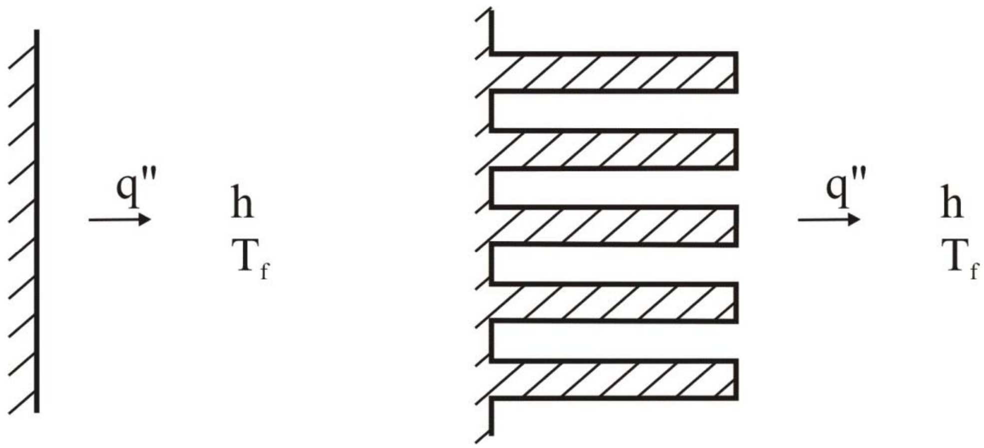
Extended Surfaces or Fins

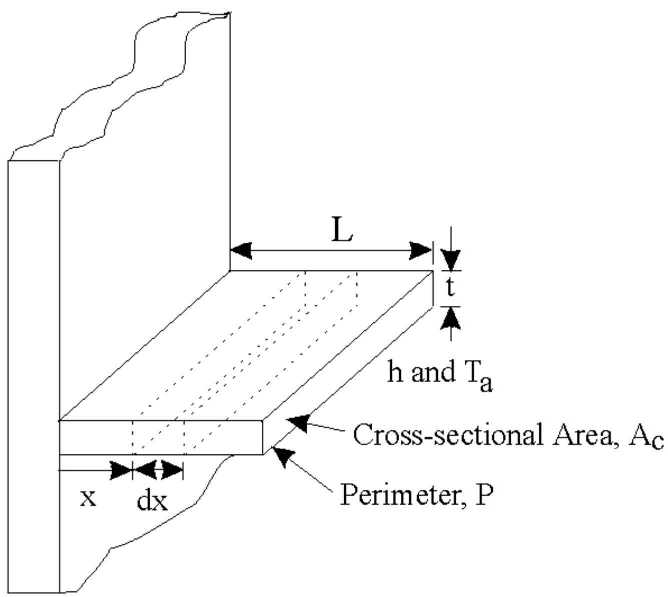
- Newton's Law of Cooling suggests that increasing heat transfer surface area will increase heat transfer rate.

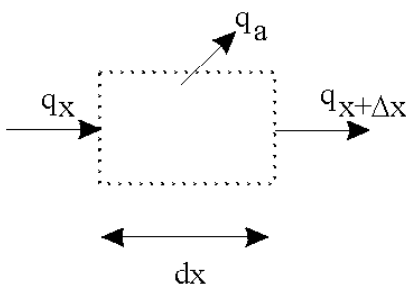
$$\dot{q} = hA\Delta T \qquad \dot{q} = \frac{\Delta T}{R} \qquad R = \frac{1}{hA}$$



- Fin temperature approaches the fluid temperature as the fin extends into the fluid.
- Heat transfer is not exactly proportional to the surface area.

One Dimensional Fin Equation





- Energy balance

$$\dot{q}_x - \dot{q}_a - \dot{q}_{x+dx} = 0$$

One Dimensional Fin Equation

- Conduction (Fourier's law)

$$\dot{q}_x = -k \frac{dT_f}{dx} \qquad \dot{q}_{x+dx} = -kA \frac{dT_f}{dx} - k \frac{d}{dx}\left(A \frac{dT_f}{dx} \right) dx$$

- Convection (Newton's law of cooling)

$$\dot{q}_a = hPdx(T_f - T_a)$$

Energy balance becomes

$$\frac{d^2 T_f}{dx^2} - \frac{hp}{kA}\left(T_f - T_\infty\right) = 0$$

One Dimensional Fin Equation

Introducing

$$\theta(x) = T_f(x) - T_a$$

$$m = \sqrt{hP/kA_c}$$

Energy balance becomes

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0$$

One Dimensional Fin Equation

- Solution for the boundary condition of

$$T(x=0)=T_0 \qquad \left.\frac{dT}{dx}\right|_{x=L}=0$$

is

$$\frac{\theta(x)}{\theta_0}=\frac{\cosh[m(L-x)]}{\cosh[mL]}$$

- To determine the total heat transfer rate from the fin, one can evaluate the gradient at the fin base and substitute into Fourier’s law:

$$\dot{Q}=A_c k \theta_a m \tanh(mL)$$

One Dimensional Fin Equation

- Alternatively, the convective heat transfer can be integrated over the fin surface to determine the total heat transfer rate :

$$\dot{Q} = \int_0^L hP\,\theta(x)dx$$

Convection Heat Transfer Coefficient

- Natural convection heat transfer coefficient, *h*, for a horizontal cylinder

$$Nu_m^{1/2} = 0.6 + \frac{0.387\,Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}} \quad for\ 10^{-4} < Ra_D < 10^{12}$$

where

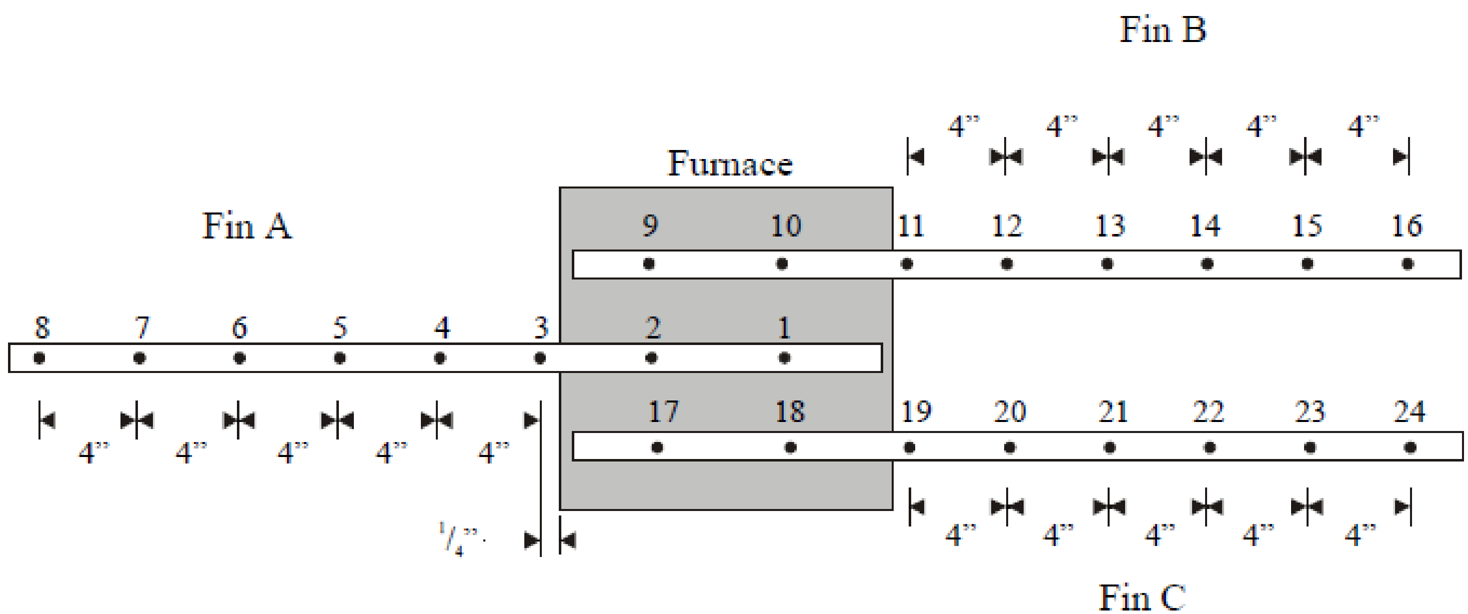
$$Nu_m \left(= hD/k_{air}\right)$$

$$Ra_D = Gr_D \Pr$$

$$Gr_D = \frac{g\beta D^3 \left(T_w - T_a\right)}{\nu^2}$$

Experiment Apparatus

- Determine the fin thermal conductivity by both methods.



(Type K Thermocouples = •)

Fin diameter : 5/8"

Length of fin A, B, and C : 23 15/16" , 23 13/16" , 23 6/16"

Determining k using Eq. 4

$$\dot{Q} = A_c k \theta_a m \tanh(mL) \qquad \text{(Eq. 4)}$$

From the curve-fit temperature profile from data, determine the heat transfer rate.

$$\dot{Q} = \int_0^L hP\,\theta(x)\,dx$$

Recall fin parameter

$$m = \sqrt{hP/kA_c}$$

Solve for k by trial-and-error method.

Determining k using Eq. 3

$$\frac{\theta(x)}{\theta_0} = \frac{\cosh[m(L-x)]}{\cosh[mL]} \qquad \text{(Eq. 3)}$$

From the temperature data, determine the fin parameter m by trial-and-error method.