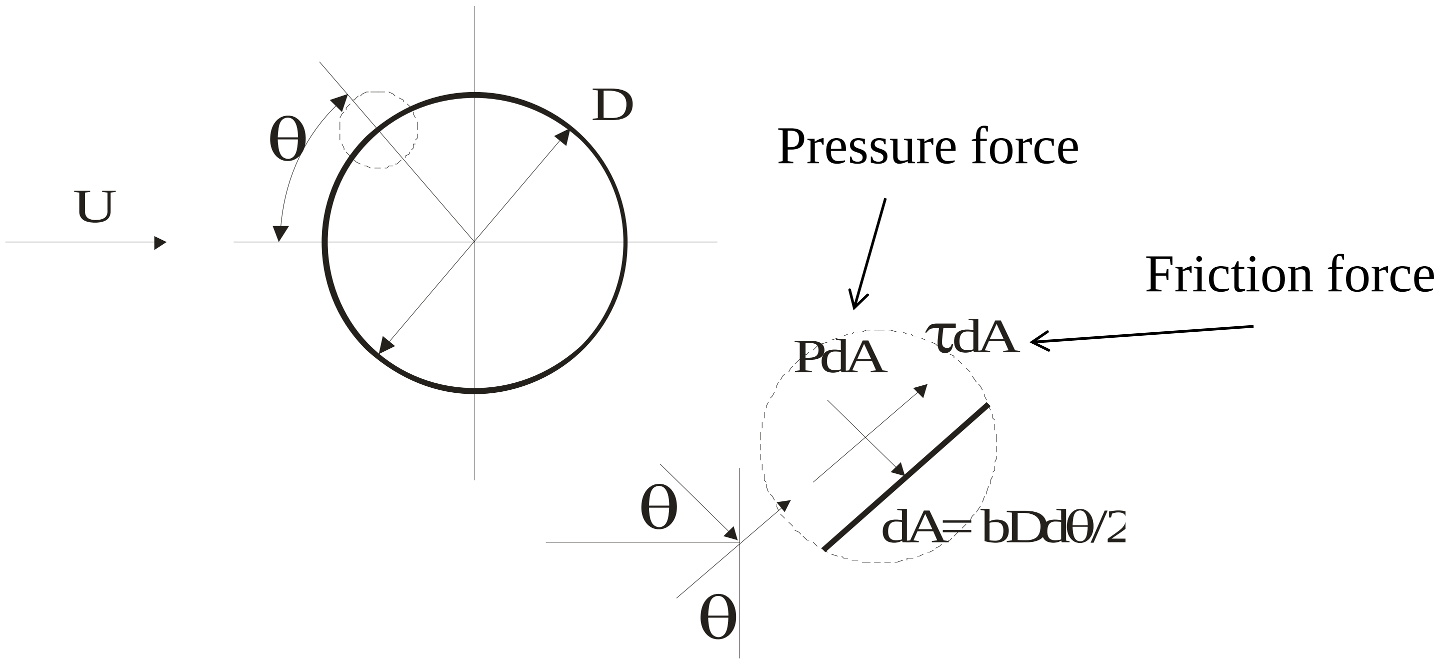
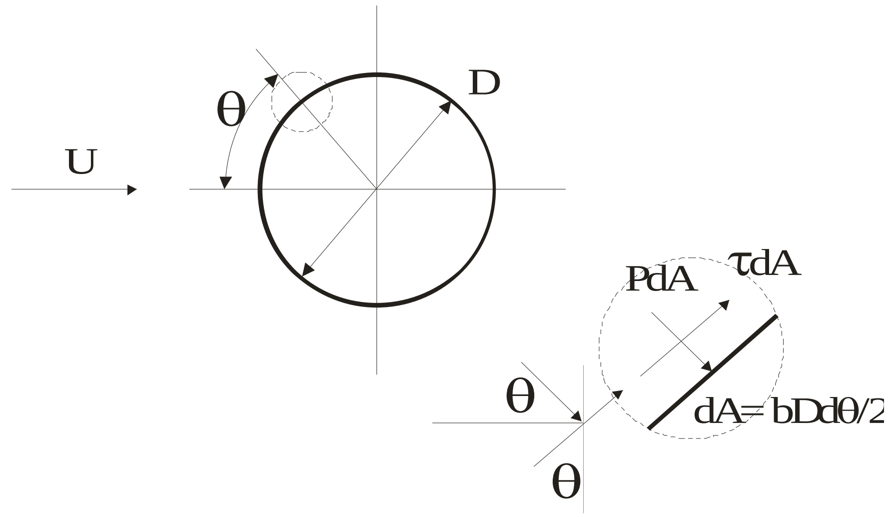
External Viscous Flow

Consider External Flow around a cylinder and the external forces acting on the surface of the cylinder in crossflow.



Friction Drag

Force in horizontal direction (fluid direction) due to fluid shear stress.



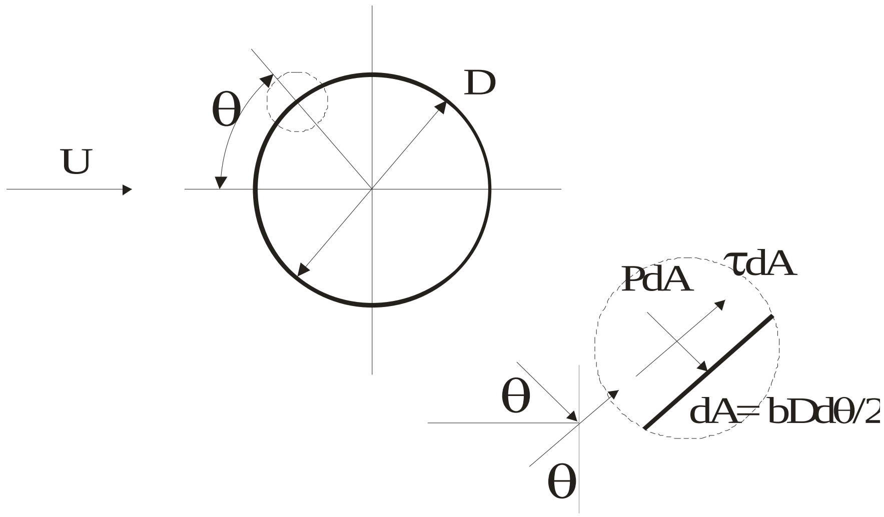
$$D_f = \int_A \tau_w \sin\theta \, dA$$

$$D_f = \int_A \tau_w \sin\theta \, b \frac{D}{2} d\theta$$

$$D_f = b \frac{D}{2} \int_A \tau_w \sin\theta \, d\theta$$

Pressure Drag

Force in horizontal direction (fluid direction) due to fluid pressure forces.



$$D_p = \int_A P\cos\theta \, dA$$

$$D_p = \int_A P\cos\theta \, b\frac{D}{2} d\theta$$

$$D_p = b\frac{D}{2}\int_A P\cos\theta \, d\theta$$

Total Drag

Total force in horizontal direction (fluid direction) due to fluid pressure forces and shear forces.

$$D_T = D_p + D_f$$

$$D_T = b\frac{D}{2}\left[\int_A P\cos\theta\, d\theta + \int_A \tau_W \sin\theta\, d\theta\right]$$

Drag Coefficient

Without detailed information, the alternative is to define a dimensionless drag coefficient.

$$C_D = \frac{D_T}{\frac{1}{2}\rho U^2 A_c}$$

where A_c is a characteristic area (usually projected area).

Drag Coefficient

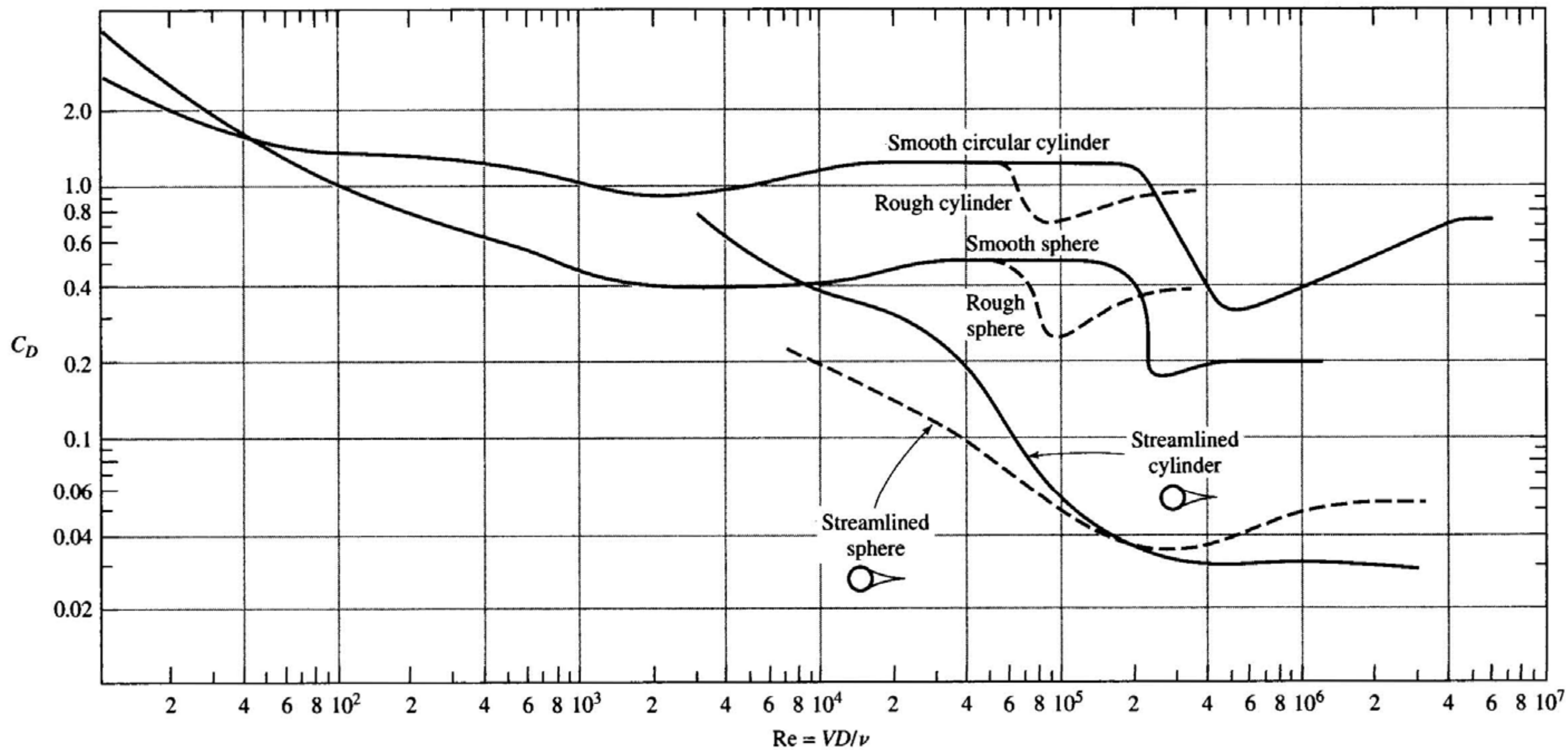
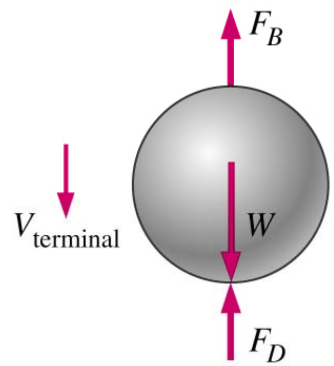


FIGURE 8.9 Drag coefficients for flow around a long cylinder and a sphere. (See E. Achenbach, *J. Fluid Mech.*, Vol. 46, 1971, and Vol. 54, 1972.)

Terminal Velocity

1. When a body is first dropped in the atmosphere or water, it will accelerate under the action of its weight.
2. As the speed of the body increases, the drag force will increase.
3. Finally the drag will reach a magnitude such that the sum of all the external forces on the body will be zero.
4. Acceleration will cease and the body will have attained its terminal velocity.



$F_D = W - F_B$
(No acceleration)

Example

Problem

Determine the terminal velocity of a 30-cm-diameter smooth sphere (s.g. = 1.02) if it is dropped in water at 20 C.

Solution

$$+\uparrow \sum F_z = 0$$

$$F_{drag} - F_{weight} + F_{bouyant} = 0$$

$$\gamma_{sphere}\left(\frac{4}{3}\pi R^3\right) = C_D \frac{1}{2}\rho V^2\left(\pi R^2\right) + \gamma_{water}\left(\frac{4}{3}\pi R^3\right)$$

$$V = \left[\frac{8R(s.g. - 1)\gamma_{water}}{3\rho C_D}\right]^{1/2} = \frac{0.28}{\sqrt{C_D}}$$

Example

Guess value of drag coefficient

$$C_D = 0.5$$

$$V = \frac{0.28}{\sqrt{C_D}} = 0.40\ \text{m/s}$$

$$Re_D = \frac{VD}{\nu} = 1.2 \times 10^5$$

Read drag coefficient from chart at this Reynolds number.

Drag Coefficient

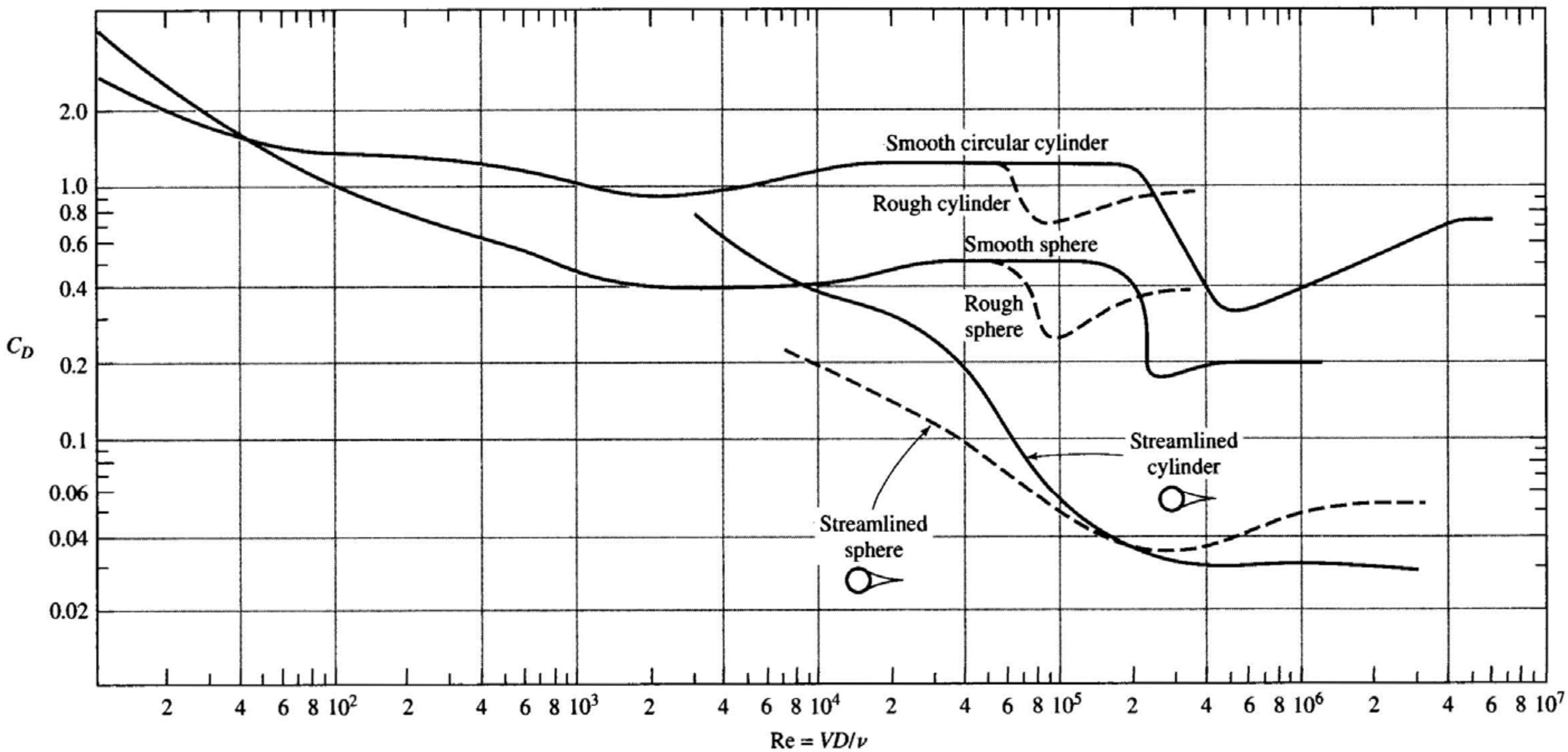


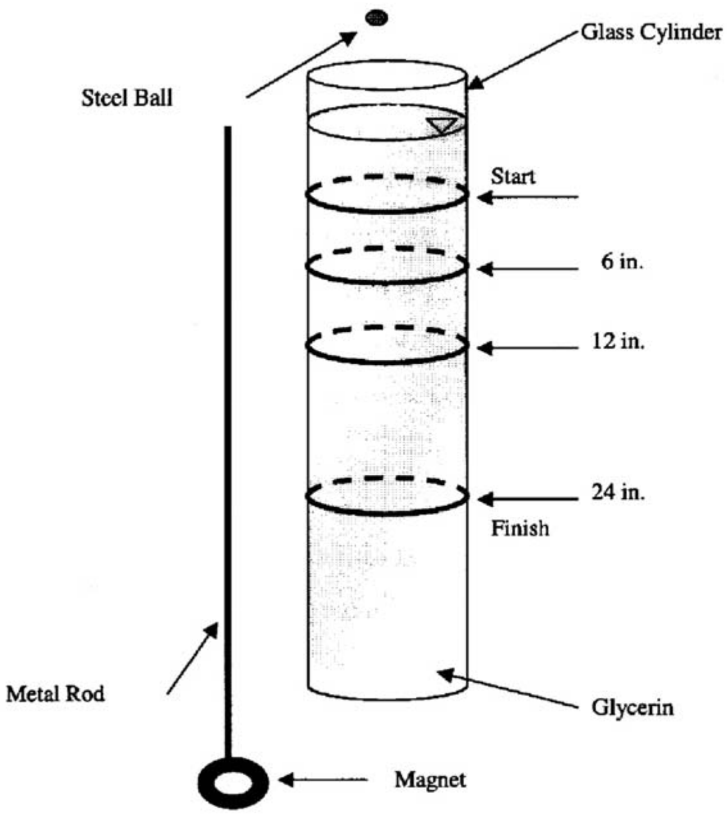
FIGURE 8.9 Drag coefficients for flow around a long cylinder and a sphere. (See E. Achenbach, *J. Fluid Mech.*, Vol. 46, 1971, and Vol. 54, 1972.)

Example

The drag coefficient at this Reynolds number is 0.5 which is equal to the guessed value.

External Flow Experiment

With an understanding of drag force and terminal velocity, the viscosity of a fluid may be measured by a falling-ball method.



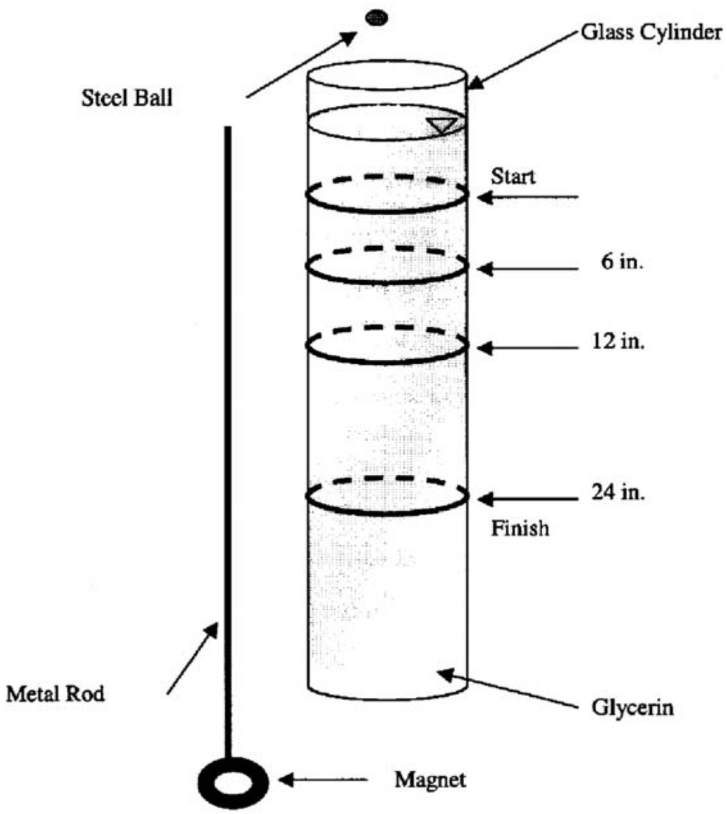
From the terminal velocity example we demonstrated that for fixed ball and fluid conditions,

$$V_{terminal} = f(Re)$$

So with known terminal velocity we can determine the fluid viscosity.

External Flow Experiment

Determine terminal velocity of ball in liquid and compare to predicted value.

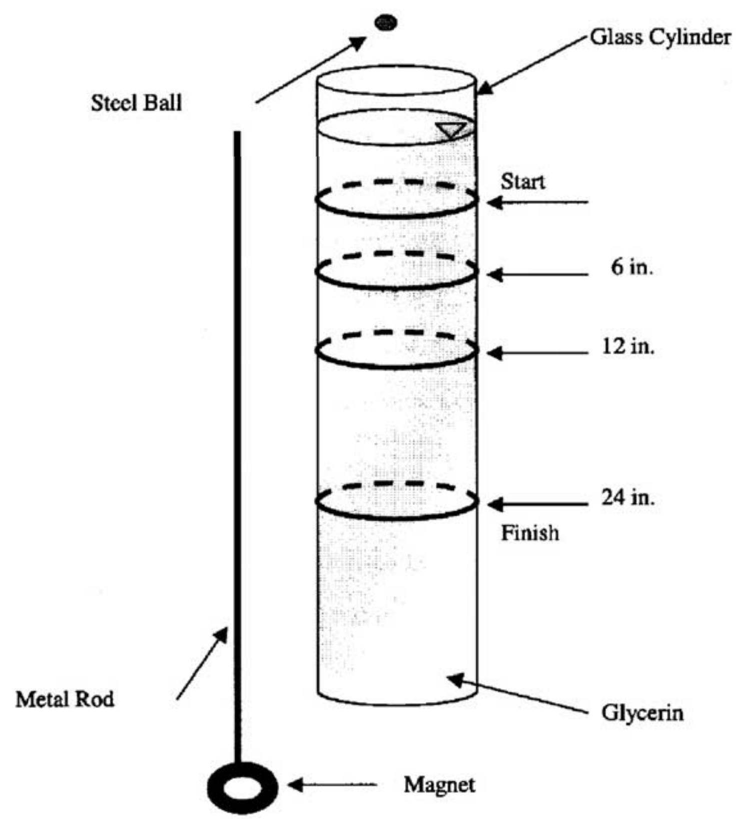


- Drop ball in column of glycerin.
- Start stopwatch when ball reaches 12 in mark.
- Stop stopwatch when ball reaches 24 in mark.

External Flow Experiment

If the ball has reached terminal velocity, then it may be determine from the experimental data.

$$V = \frac{\text{distance travelled}}{\text{duration of time}}$$



External Flow Experiment

- Experiment is dependent on terminal velocity assumption.
- An estimate of whether ball has reached terminal velocity would be useful.

$$+\uparrow \sum F_z = m\frac{dV}{dt}$$

$$F_{drag} - F_{weight} + F_{bouyant} = m\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{1}{m}\left(C_D \frac{1}{2}\rho V^2 A_{fr} - mg + \gamma_{water}\forall\right)$$