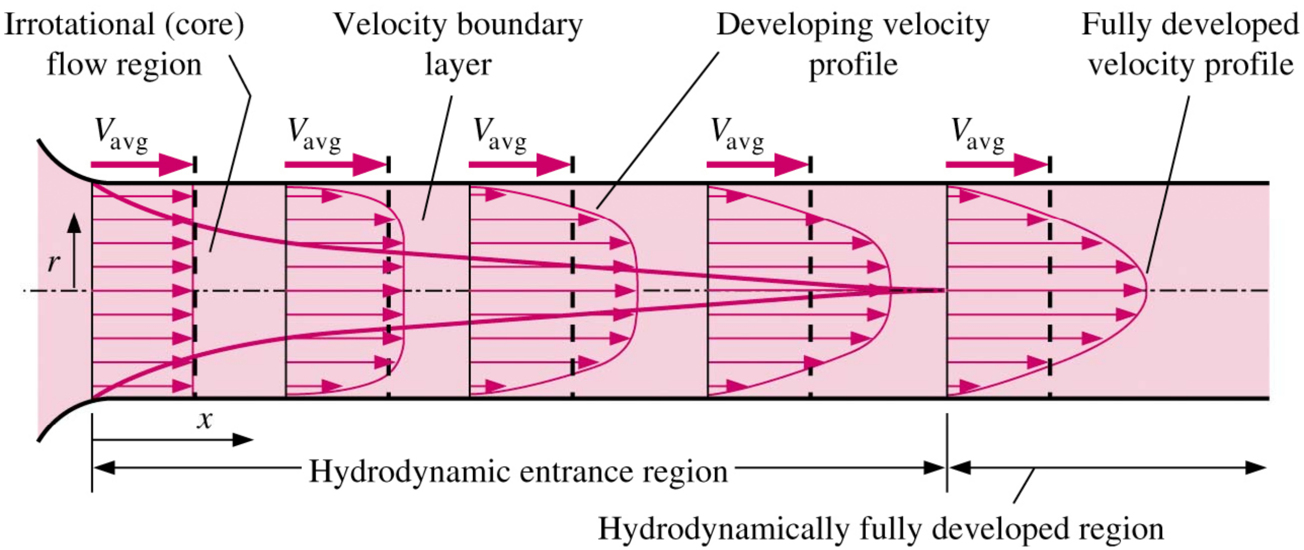
Internal Viscous Flow in Pipes

- The purpose of this lab is to demonstrate internal flow in a pipe.
- Region near the entrance of pipe, boundary layer grows until its thickness is equal to half the tube diameter.



$$\frac{L_e}{D} \sim Re_D$$

Laminar versus Turbulent

- Criterion for laminar or turbulent flow
  - Laminar $\mathrm{Re}_D < 2000$
  - Transition Region $2000 < \mathrm{Re}_D < 4000$
  - Turbulent $\mathrm{Re}_D > 4000$
- Typically
  - Laminar $\mathrm{Re}_D < 2300$
  - Turbulent $\mathrm{Re}_D > 2300$

Entrance Length

- Laminar flow
  - Hydraulic $$\frac{L_{e,h}}{D} \approx 0.05\,\mathrm{Re}_D$$
  - Thermal $$L_{e,t} \approx \mathrm{Pr}\, L_{e,h}$$
- Turbulent flow $$L_{e,t} \approx L_{e,h} \approx 10D$$

Pressure Drop Equations

- Energy equation between two locations in a constant cross-sectional pipe
- fully developed
- steady-state, steady-flow
- adiabatic

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2 + h_L$$

- The head loss is defined as

$$h_L = \frac{fL}{D}\frac{V^2}{2g}$$

Pressure Drop Equations

- Combining two equations

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2 + \frac{fL}{D}\frac{V^2}{2g}$$

- Mass balance between two locations

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$V_1 = V_2$ (for constant density and cross-sectional area)

Pressure Drop Equations

- The energy equation

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D}\frac{V^2}{2g}$$

- Reduces to

$$\frac{P_1 - P_2}{\gamma} = \frac{fL}{D}\frac{V^2}{2g}$$ (Darcy-Weisbach equation)

Friction factor

- Correlations
  - Laminar flow

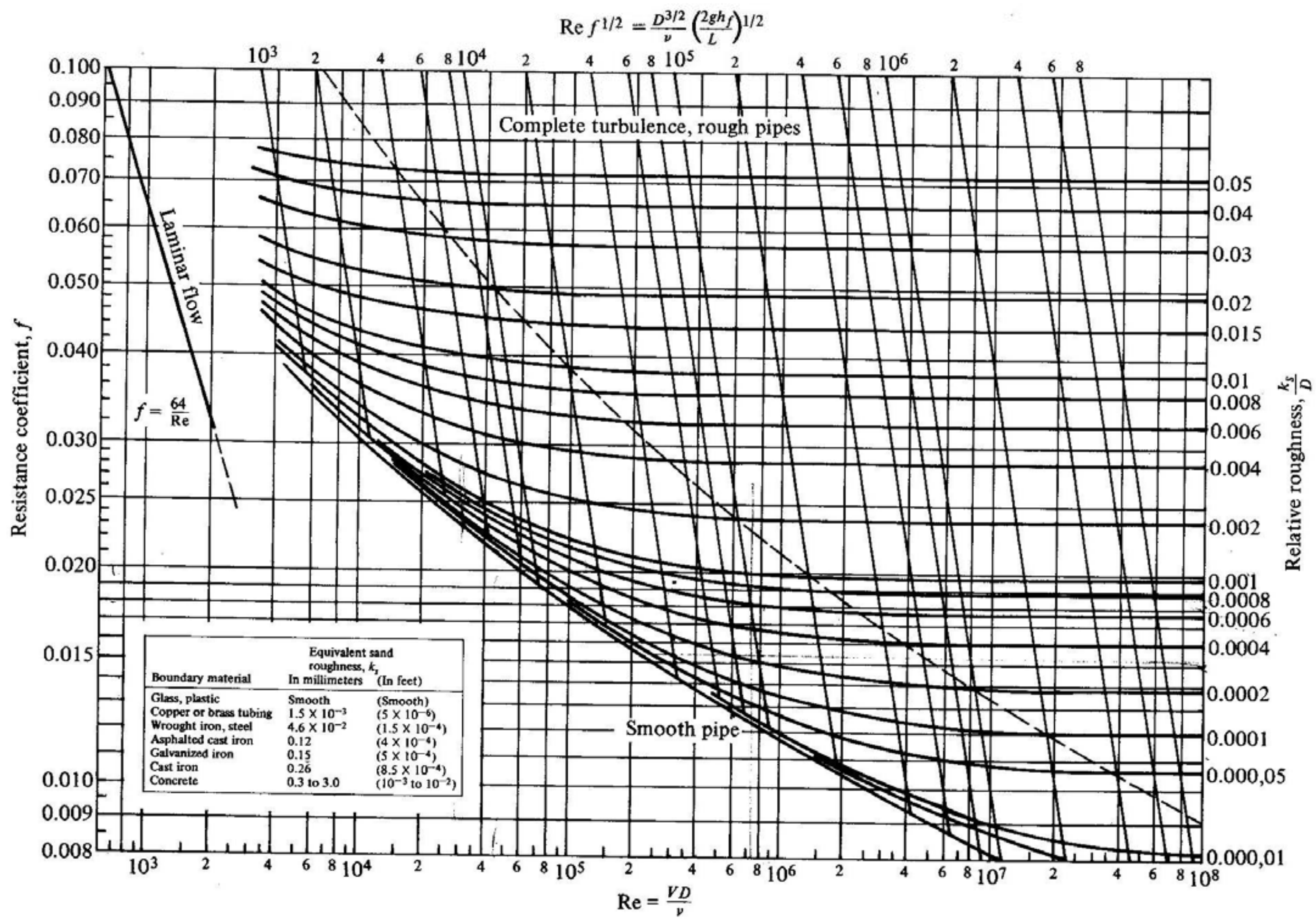
$$f = \frac{64}{Re_D}$$

  - Turbulent flow, smooth pipes

$$f = \left(1.82 \log Re_D - 1.64\right)^{-2} \qquad \text{for} \qquad Re_D > 10^4$$

- Moody Chart

Moody Chart



Boundary material	Equivalent sand roughness, k_s In millimeters	(In feet)
Glass, plastic	Smooth	(Smooth)
Copper or brass tubing	1.5×10^{-3}	(5×10^{-6})
Wrought iron, steel	4.6×10^{-2}	(1.5×10^{-4})
Asphalted cast iron	0.12	(4×10^{-4})
Galvanized iron	0.15	(5×10^{-4})
Cast iron	0.26	(8.5×10^{-4})
Concrete	0.3 to 3.0	$(10^{-3}$ to $10^{-2})$

FIGURE 10-8 Resistance coefficient f versus Re. Reprinted with minor variations. [After Moody (18). Reprinted with permission from the A.S.M.E.]

Example

Problem

Oil with a density of 900 kg/m^3 and viscosity 0.00001 m^2/s, flows at 0.2 m^3/s through 500 m of 200 mm-diameter cast iron pipe. Determine the head loss and pressure drop if the pipe slopes down at 10^o in the flow direction.

Solution

Compute the velocity.

$$V = \frac{Q}{\pi R^2} = 6.4\, \text{m/s}$$

Reynolds number

$$Re_D = \frac{Vd}{\nu} = 128{,}000$$

Example

Relative roughness for cast iron pipe

$$\frac{\varepsilon}{D} = \frac{0.26\,\text{mm}}{200\,\text{mm}} = 0.0013$$

Moody chart

$$f \approx 0.0255$$

Head loss

$$h_L = \frac{fL}{D}\frac{V^2}{2g} = 117\,\text{m}$$

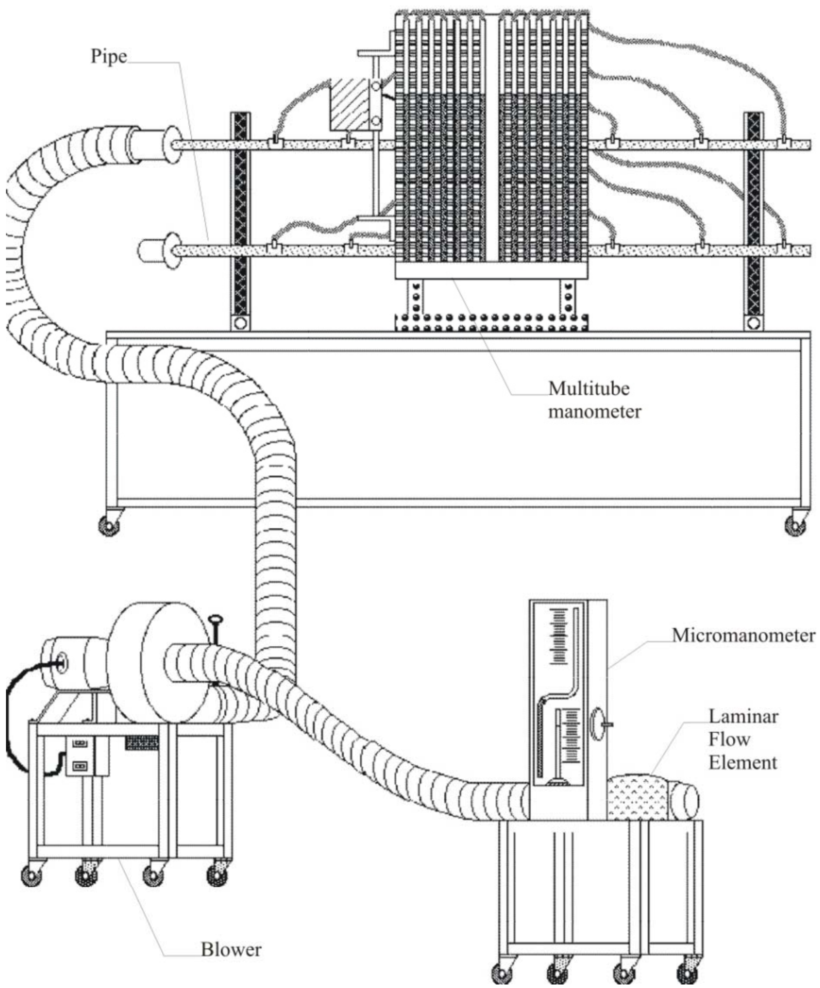
Pressure drop

$$\frac{\Delta P}{\rho g} + (z_1 - z_2) = \frac{\Delta P}{\rho g} + L\sin\theta = h_L$$

$$\Delta P = 265{,}000\,\text{Pa}$$

Internal Flow Experiment

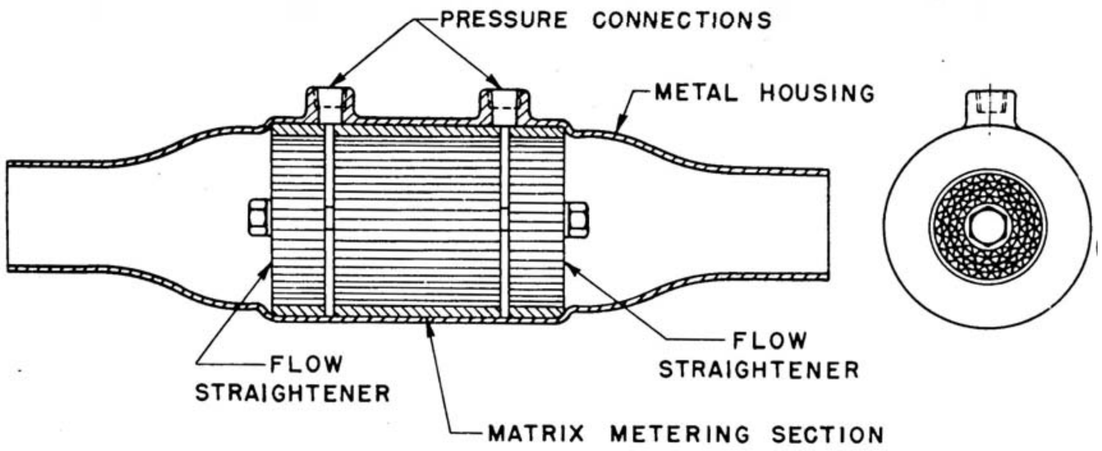
Measure pressure drop through pipe.

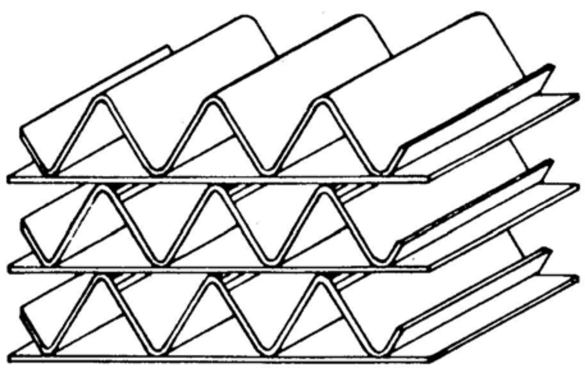


Internal Flow Experiment

Predict pressure drop through pipe.

- Measure flow rate (laminar flow element).





Laminar Flow Element

Passages are small such that flow is laminar.

$$\frac{P_1 - P_2}{\gamma} = \frac{fL}{D}\frac{V^2}{2g}$$

Passages are small such that flow is laminar.

$$f = \frac{64}{Re_D} = \frac{64\mu}{\rho VD}$$

$$\frac{P_1 - P_2}{\gamma} = \frac{64\mu}{\rho VD}\frac{L}{D}\frac{V^2}{2g}$$

$$\Delta P = \frac{64\mu}{\rho VD}\frac{L}{D}\frac{V^2}{2g}\gamma = \frac{64\mu}{\rho VD}\frac{L}{D}\frac{V^2}{2g}\rho g$$

Laminar Flow Element

$$\Delta P = \frac{64\mu}{D}\frac{L}{D}\frac{V}{2}$$

$$\Delta P = \left(\frac{32}{g}\right)(\gamma \nu)\left(\frac{L}{D^2}\right)V$$

Pressure drop is linear with fluid velocity.