5.1 Extended Surfaces

5.1.1 Introduction

With an understanding of the fin equation, you will determine the thermal conductivity of three pin fins and compare to the thermal conductivity in data tables.

5.1.2 One-dimensional Fin Equation

In order to increase heat transfer from a surface to the fluid, there are three ways: (a) increase the heat transfer coefficient (fluid properties and surface geometry), (b) increase the temperature difference between the fluid and surface, and (c) increase the surface area. Adding extended surfaces, commonly referred to as fins, can increase the surface area.

Figure 1(a) is a rectangular fin added to a surface to increase the heat transfer. The fin's length is L and its thickness is t. The cross-sectional area is A_c and the perimeter of the cross-section is P. The fin is surrounded by a fluid at temperature T_a and the heat transfer coefficient is h. A differential element of the fin of length dx is considered. Figure 1(b) is a larger view of the element and it also shows the heat transfers. Energy conducted through the fin are $\dot{q}_x$ and $\dot{q}_{x+\Delta x}$. Energy convected away from the fin by the fluid is q_a. The steady state energy balance on the element on a rate basis is

$$\dot{q}_x - \dot{q}_a - \dot{q}_{x+dx} = 0 \tag{1}$$

Using Fourier's law for the conduction terms and Newton's law of cooling for the convection term, the equation reduces to the one-dimensional, steady state, constant thermal conductivity fin equation.

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \tag{2}$$

where $\theta(x) = T_f(x) - T_a$ and $m = \sqrt{hP/kA_c}$.

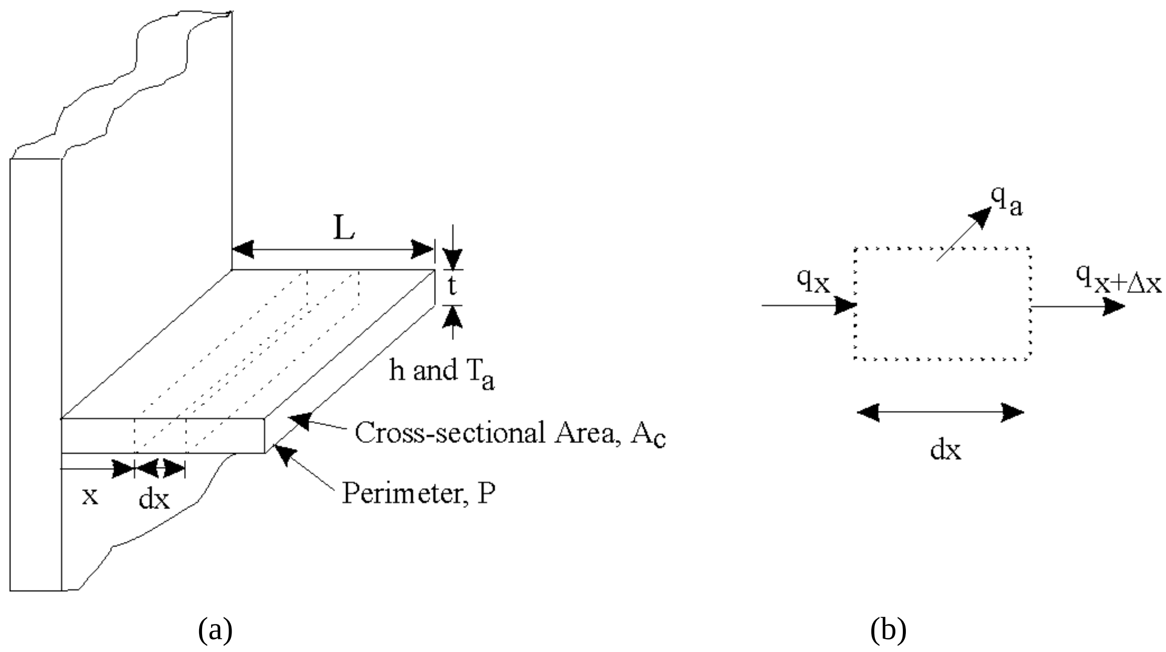


Figure 1: (a) Schematic of fin. (b) Energy balance on fin differential element.

Integrating Eq. 2 will provide the fin temperature distribution. First the boundary conditions must be defined. If the fin is sufficiently long, the heat loss through the fin tip is negligible, or the temperature gradient at L is zero. The second boundary condition is usually the known fin base temperature, T_o. The solution of the differential equation is then

$$\frac{\theta(x)}{\theta_o} = \frac{cosh[m(L-x)]}{cosh[mL]} \tag{3}$$

Using Eq. 3 the heat flow rate from the entire fin is

$$\dot{Q} = A_c k \theta_o m \tanh[mL] \tag{4}$$

In the experiment that you will run, the fin is a long cylinder. In order to evaluate the heat transfer rate, the natural convection heat transfer coefficient must be evaluated. Most undergraduate heat transfer texts have correlations for long horizontal cylinders[1].

For isothermal horizontal cylinders, Churchill and Chu proposed the following correlation

$$Nu_m^{1/2} = 0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}} \qquad for\ 10^{-4} < Ra_D < 10^{12} \tag{5}$$

[1] Ozisik,M.N., *Heat Transfer, A Basic Approach*, McGraw-Hill, New York, 1985, pp.443-445.

Where $Nu_m(= hD / k_{air})$ is the mean Nusselt number based on the cylinder diameter and Ra_D is the Rayleigh number based on the cylinder diameter. The Rayleigh number is defined as

$$Ra_D = Gr_D \, Pr \tag{6}$$

Where Pr is the Prandtl number and Gr_D is the Grashof number based on the cylinder diameter defined as

$$Gr_D = \frac{g\beta D^3 (T_w - T_a)}{\nu^2} \tag{7}$$

Where β is the volumetric coefficient of thermal expansion, g is the gravitational constant, T_w is the surface temperature, T_a is the air temperature, and ν is the air's kinematic viscosity. The volumetric coefficient can be evaluated for an ideal gas as the inverse of the absolute temperature. The Rayleigh number is the ratio of buoyancy force to the viscous force acting on the fluid.

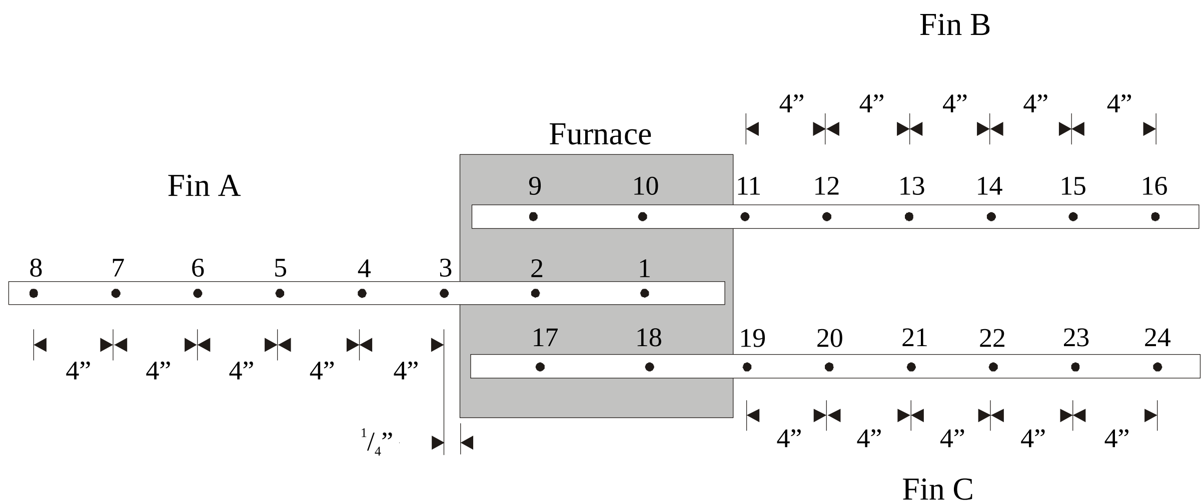
The experiment specifically involves determining the thermal conductivity of a pin fin.

5.1.3 Description of Apparatus

The apparatus consists of three cylindrical metal rods receiving heat through a constant temperature furnace as shown in Figure 2. Temperature measurements are made at the locations indicated for each rod. Furnace temperature should be preset to approximately 300 F.

5.1.4 Safety Guidelines

- Do not exceed the recommended temperature in the furnace; adjust input power to furnace accordingly.
- Do not open the furnace door while its interior temperature is above 60 C.
- Do not touch the metal rods while heated.
- Turn thermocouple dial slowly.



(Type K Thermocouples = •)

Fin diameter : $^5/_8$”

Length of fin A, B, and C : 23 $^{15}/_{16}$” , 23 $^{13}/_{16}$” , 23 $^6/_{16}$”

Figure 2: Test Set-up.

Extended Surface Worksheet

1. List the objectives of the experiment.

2. Obtain air properties: μ, k, α, Pr, etc.
3. Select one of the rods to study for the experiment.
4. Calculate the air average heat transfer coefficient for the heated rod.

   Calculation:

Heat transfer coefficient __________________ W/m^2-K

For the selected rod do the following:

5. Determine k by the heat transfer rate method.

a) Curve fit the temperature distribution $\theta(= T_w - T_a)$ along each rod as

$$\theta(x) = a + bx + cx^2 + dx^3 + ex^4 \tag{8}$$

a = ____________________

b = ____________________

c = ____________________

d = ____________________

e = ____________________

b) Substitute $\theta(x)$ into the following equation and integrate to find the heat transfer rate.

$$\dot{Q} = \int_0^L hP\,\theta(x)dx \qquad (9)$$

Where P is the fin perimeter.

Calculation:

c) Using Eq. 4, determine k.

Thermal conductivity _________________ W/m-K

5. Determine k by the temperature method. Using the fin equation that assumes negligible heat loss at tip, Eq. 3, solve for the value of k.

Thermal conductivity _________________ W/m-K

Discussion of Results and Conclusion