4.1 External Flow

4.1.1 Introduction

The drag force due to external flow over bodies is commonly written in terms of a drag coefficient as

$$F_D = C_D \frac{1}{2g_c} \rho V^2 \tag{1}$$

Where V is the approaching fluid velocity and ρ is the approaching fluid density. A is a characteristic area usually taken as the projected frontal area. The constant C_D is known as the drag coefficient.

For incompressible flow, the drag coefficient is a function of the body shape and the Reynolds number. Figure 1 shows the variation of C_D with Re for spheres. From this figure, it is seen that C_D is approximately constant at 0.47 for the Reynolds number range of 10^3 to 10^5. For other body shapes, the C_D variation with Re have the same characteristics, namely, nearly constant for a certain range of Reynolds number.

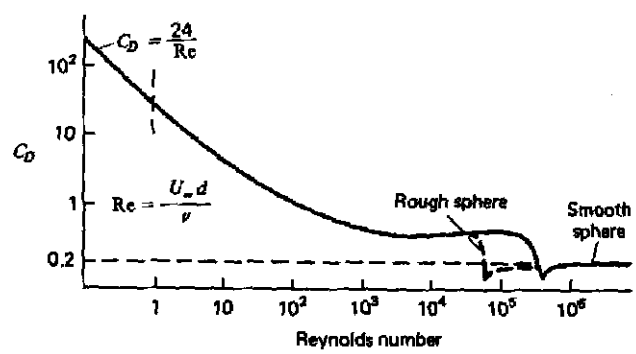

Figure 1: Drag coefficient for flow around a sphere.[1]

The objective of this experiment is to use your understanding of external flow around a sphere, to determine the viscosity of a fluid. The viscosity of the fluid will be measured by the falling sphere method.

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[1] Potter, M.C., and J.F. Foss, Fluid Mechanics, Great Lakes Press, Inc., p. 458, 1982.

The falling sphere experiment is an interesting method for determining the dynamic viscosity, μ, of a fluid. The fluid of unknown viscosity is placed into a tall cylindrical container. Spheres of known size and density are allowed to drop through the fluid. After the sphere has reached termina1 ve1ocity, the distance it falls and the time required to cover this distance is measured. With this information and the diameter and density of the sphere and density of the oil, the viscosity of the oil can be calculated.

For a solid sphere falling under gravity, the viscosity can be calculated by measuring the terminal ve1ocity of the sphere. A force balance is used to derive the needed theoretical relation:

$$\text{weight of sphere} = \text{drag force} + \text{buoyant force} \tag{1}$$

Where

$$\text{weight of sphere} = \frac{4}{3}\pi r_s^{\,3}\rho_s g \tag{2}$$

$$\text{drag force} = C_{\mathrm{D}}\rho_f \frac{V_o^{\,2}}{2}A = C_{\mathrm{D}}\rho_f \frac{V_o^{\,2}}{2}\pi r_s^{\,2} \tag{3}$$

$$\text{buoyant force} = \frac{4}{3}\pi r_s^{\,3}\rho_f g \tag{4}$$

and r_s, ρ_s, ρ_f ,V_o, A, and C_D are, respectively, the radius of the sphere, the density of the sphere, the density of the fluid, the terminal velocity, the projected area of the sphere, and the drag coefficient.

For laminar f1ows, the drag coefficient is given by:

$$\mathrm{C_D} = \frac{24}{\mathrm{Re}} = \frac{24\mu_f}{\rho_f V_o 2r_s} \tag{5}$$

Where *Re* and μ_f are the Reynolds number and the dynamic viscosity, respectively. Therefore, Eq. (1) becomes:

$$\frac{4}{3}\pi r_s^{\,3}\rho_s g = \frac{24\mu_f}{\rho_f V_o 2r_s}\frac{\rho_f V_o^{\,2}}{2}\pi r_s^{\,2} + \frac{4}{3}\pi r_s^{\,3}\rho_f g \tag{6}$$

Which can be solved for the viscosity as:

$$\mu_f = \frac{2r_s^{\,2}\left(\rho_s - \rho_f\right)g}{9V_o} \tag{7}$$

The above equation is quite useful in that it is simple to use for calculation purposes. However, it does not account for the effect of the tube wall. In order to account for this, the modifications are made. First, the drag coefficient is modified as**:**

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + 2.1\frac{r_s}{r_t}\right) \tag{8}$$

Where r_t is the radius of the tube. Second the corrected terminal velocity is given by:

$$V_c = V_o\left(1 + \frac{9}{4}\frac{r_s}{r_t} + \left(\frac{9}{4}\frac{r_s}{r_t}\right)^2\right) \tag{9}$$

Now, by including the tube wall effects, the following equation results:

$$\mu_f = \frac{2r_s^{\,2}\left(\rho_s - \rho_f\right)g}{9V_o\left(1 + 2.1\frac{r_s}{r_t}\right)\left(1 + \frac{9}{4}\frac{r_s}{r_t} + \left(\frac{9}{4}\frac{r_s}{r_t}\right)^2\right)} \tag{10}$$

This equation should more accurately predict the viscosity of a fluid tested by the falling sphere method. The equations derived above, (1) – (9), are in correct form for most measurement systems.

4.1.2 Description of Apparatus

The test setup that you will use is shown in Fig. 2. It consists of a long, vertical cylindrical tube, with diameter 83 mm and length 1.25 m. It is filled with the viscous fluid Glycerin. Note the rings on the tube identify significant locations that should be labeled. A steel ball will be dropped in the liquid and timed with a stopwatch, which is not shown. Once the time is recorded, the magnet is used for the retrieval of the metal ball.

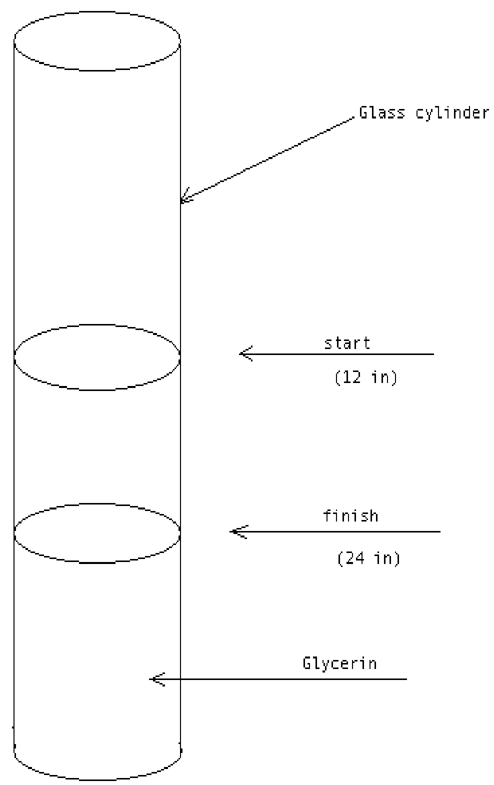

Figure 2: Experimental Apparatus.

External Flow Worksheet

1. List objectives of this experiment.

2. Measure necessary variables such as sphere diameter.

3. Measure amount of time for the steel ball to travel from 12 in mark to 24 in mark.

4. Perform experiment several times.

5. Determine the experimental viscosity.

   Calculation:

Experimental Viscosity ________________

6. Compare experimental viscosity with expected value.

   Calculation:

% Difference ________________

Discussion of Results and Conclusions