Kinematic Viscosity

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Group 1

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Abstract

The purpose of this experiment was to explore Poiseuille's law to determine the kinematic viscosity of water. This was done by using an Ubbelohde glass viscometer (PSL Rheotek, Essex, England) and examining the nature of the viscometer coefficient. Data were collected at 23.5±1.8°C; the mean kinematic viscosity was 0.974±0.078 cSt with a range of 0.969±0.077 to 0.979±0.078 cSt. The theoretical kinematic viscosity at this temperature was 0.927 cSt. The mean Reynolds number of the water flowing through the capillary was 42.5±3.5 and ranged from 42.0±3.5 to 42.9±3.6.

Objective

The objective of this laboratory experiment was to demonstrate Poiseuille's law and use the relation to calculate the kinematic viscosity of water. This was done by using an Ubbelohde glass viscometer (PSL Rheotek, Essex, England) and measuring the time for water to flow a specified distance in a small capillary. The theory behind capillary viscometers was also examined, especially the nature of the viscometer constant.

Theory

The ratio between dynamic viscosity μ and density ρ is referred to as kinematic viscosity ν, expressed as

$$\nu = \frac{\mu}{\rho} \quad [1]$$

Using the theoretical relation of a differential fluid element flowing through a long circular pipe during laminar, fully developed flow, the kinematic viscosity can be determined in the Ubbelohde viscometer. For laminar flow, i.e. $Re \lesssim 2300$, the pressure loss in the capillary section of the viscometer is

$$\Delta P = P_1 - P_2 = \frac{32\mu L V_{avg}}{D^2} \quad [2]$$

Where P is pressure, μ is dynamic viscosity, L is the length of the capillary, V_{avg} is the average velocity of the fully developed velocity profile of the fluid, and D is the diameter of the capillary. Equation 2 can be rearranged for V_{avg} as follows

$$V_{avg} = \frac{\Delta P D^2}{32\mu L} \quad [3]$$

And the volumetric flowrate $\dot{V}$ is

$$\dot{V} = V_{avg} A_c \quad [4]$$

Where A_c is the cross-sectional area of the capillary. Equation 3 may be substituted into Equation 4 to yield

$$\dot{V} = V_{avg}A_c = \frac{\Delta P D^2}{32\mu L}A_c = \frac{\Delta P D^2}{32\mu L} \cdot \frac{\pi}{4}D^2 \qquad [5]$$

This equation results in the thermal fluid principle for volumetric flowrate known as Poiseuille's law

$$\dot{V} = \frac{\Delta P \pi D^4}{128\mu L} \qquad [6]$$

In order to use the relation for the Ubbelohde viscometer, which employs a vertical flow capillary, the pressure differential is expressed as

$$\Delta P = \rho g h \qquad [7]$$

Where ρ is the density of the liquid, g is acceleration due to gravity, and h is the height of the liquid column. This change reflects the hydrostatic pressure within the fluid in the capillary section. The mean height of the fluid from the start time to the end time is used for the viscometer, and is measured from the bottom of the capillary.

Using Equation 1 and the relation for volumetric flowrate,

$$\dot{V} = \frac{V}{t} \qquad [8]$$

Where *V* is the volume of fluid that passes through the capillary in time *t*, the modified version of Poiseuille's law is expressed as

$$\frac{V}{t} = \frac{gh\pi D^4}{128 v L} \qquad [9]$$

Solving for kinematic viscosity, the equation is rearranged to

$$v = \frac{\pi D^4 g h}{128 L V} \cdot t \qquad [10]$$

Noting that the values multiplied by *t* are the geometric constraints of the viscometer and do not change throughout the experiment, a constant C is developed

$$C = \frac{\pi D^4 g h}{128 L V} \qquad [11]$$

Therefore, the kinematic viscosity of a fluid can be calculated knowing the viscometer constant and multiplying by the efflux time

$$v = C \cdot t \qquad [12]$$

Experimental Apparatus

The Ubbelohde viscometer is a U-shaped glassware instrument used to measure the kinematic viscosity of a liquid (Figure 1). It contains three tubes; tube 1 has a storage reservoir (#4 as seen in Figure 1) into which a fluid is poured until the liquid meniscus is between the two fill lines (10) in the fluid reservoir. Tube 2 contains a capillary (7), a measuring sphere (8) and a pre-run sphere (9); it extends upward from the leveling bulb (6). The capillary restricts flow so that the fluid flows at a measurable rate. Tube 3 extends upward from below the capillary in the leveling bulb, and runs parallel to tube 2. The purpose of tube 3 is to equilibrate the pressure above the surface of the fluid and below the fluid (atmospheric pressure). By equilibrating these two pressures, the pressure in the fluid column is made equivalent to the hydrostatic pressure of the fluid within and above the capillary. A stopwatch was used to measure the efflux time.

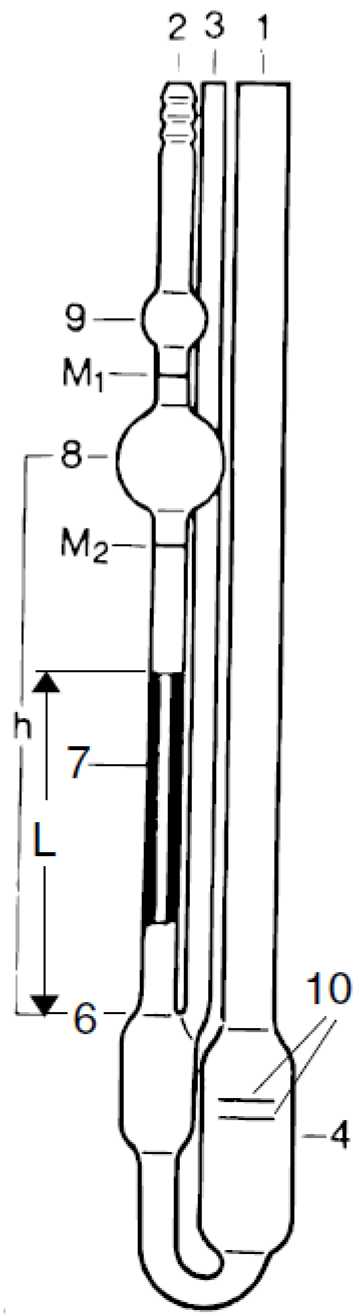

Figure 1 – Ubbelohde viscometer

Experimental Procedure

Prior to starting the experiment, the viscometer was charged with 15 mL of distilled water using a graduated cylinder. A thermocouple was placed in the reservoir of the viscometer and the system was allowed to thermally equilibrate with room temperature over the course of 20 min. Using a small epoxy syringe and surgical tube, the water was drawn up tube 2 of the viscometer while sealing tube 3. The vacuum was applied until the test fluid halfway filled bulb 9 of the viscometer. The vacuum was removed, and the efflux time was measured from line M_1 to M_2 and data recorded. The efflux time was multiplied by the viscometer constant to find the kinematic viscosity of water.

Discussion of Results

Five trials were conducted at 23.5±1.8°C to determine the kinematic viscosity of water. The average kinematic viscosity at this temperature was 0.974±0.078 cSt with a range of 0.969±0.077 to 0.979±0.078 cSt. To the same number of significant figures, the theoretical kinematic viscosity at this temperature was 0.927 cSt.[1]

The mean Reynolds number of the fluid flowing through the capillary was 42.5±3.5 and ranged from 42.0±3.5 to 42.9±3.6. Because $Re \ll Re_{cr} \approx 2300$, i.e. the flow through the capillary was laminar, the Poiseuille's law derivation for this viscometer was valid.

The kinematic viscosity data were plotted in Figure 2 along with the theoretical value for water at 23.5°C. As can be seen from the plot, the measured values match the theoretical value within the bounds of uncertainty. However, the measured value for each trial was higher than the theoretical value. This implies that the actual test temperature was lower than the recorded test temperature, and is most likely due to a bias error in the temperature measurements. If the viscometer was used as intended, i.e. with a precision controlled temperature bath, then the experimental data would most likely be closer to the theoretical data. The temperature bath recommended by the manufacturer controls the temperature to ±0.02°C. Under these conditions, the kinematic viscosity uncertainty would be ±0.19%, which is much smaller than the kinematic viscosity uncertainty in this experiment (±8%). For a further explanation of this uncertainty, see the Uncertainty section in the Appendix. Other sources of uncertainty may be dust contamination in the fluid, which could increase the viscosity reading. Also, the temperature was only recorded before the first test. If the temperature fluctuated between trails, then the viscosity would also fluctuate between trials as it did in the data. Kinematic viscosity has a strong dependence on temperature.

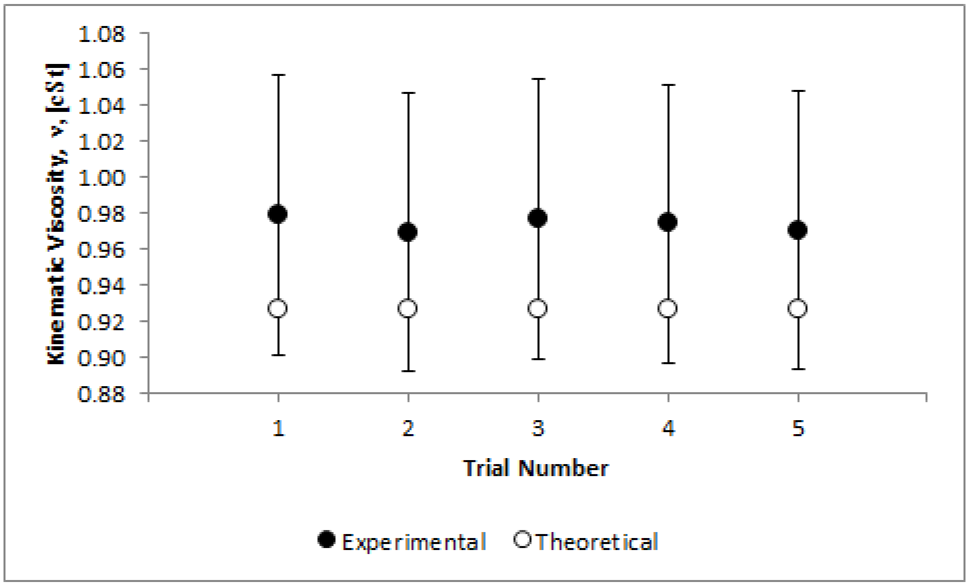

Figure 2 - Experimental results and the theoretical values for the kinematic viscosity of water for five trials.

Conclusion

The objectives of the experiment to demonstrate Poiseuille's law and use the relation to calculate the kinematic viscosity of water were satisfied using the Ubbelohde viscometer. The measured viscosity of water matched the theoretical value. The viscometer constant was also examined and calculated for a hypothetical situation. By using Poiseuille's law and conducting the experiment, the student developed an understanding of the geometric construction and operating theory of the Ubbelohde viscometer.

References

[1]Çengel, Y. A., & Cimbala, J. M. (2010). *Fluid mechanics, fundamentals and applications*. (2nd ed.). Boston: McGraw-Hill.

APPENDIX

Raw Data

Table A1 – Raw data

Raw Data					
Trial	**Time (s)**	**Kinematic Viscosity (cSt)**	**Volumetric Flow Rate (m^3/s)**	**Velocity (m/s)**	**Re**
1	334.330	0.9793	1.486×10^{-8}	8.949×10^{-2}	42.01
2	330.971	0.9694	1.502×10^{-8}	9.035×10^{-2}	42.87
3	333.464	0.9767	1.490×10^{-8}	8.967×10^{-2}	42.23
4	332.510	0.9739	1.495×10^{-8}	8.993×10^{-2}	42.48
5	331.343	0.9705	1.500×10^{-8}	9.025×10^{-2}	42.78

Sample Calculations

The equation derived from Poiseuille's law that solves for kinematic viscosity is

$$v = \frac{\pi D^4 gh}{128LV} \cdot t$$

Where v is kinematic viscosity, D is the diameter of the capillary, g is the acceleration due to gravity, h is the mean pressure height, L is the length of the capillary, and V is the volume that passes through the capillary during time t. Rearranging this equation to solve for volume,

$$V = \frac{\pi D^4 gh}{128Lv} \cdot t$$

Plugging in the given information about the viscometer and the properties of methanol, the volume can be calculated.

$$V = \frac{\pi (0.4600mm)^4 \left(9.81\frac{m}{s^2} \cdot \frac{1000mm}{1m}\right)(120.84mm)}{128(89.5mm)\left(0.7429\frac{mm^2}{s}\right)} \cdot 253.64s$$

$$V = 4970mm^3 = 4.970mL$$

Now that the volume is known, the viscometer constant can be calculated.

$$C = \frac{\pi D^4 gh}{128LV}$$

$$C = \frac{\pi(0.4600mm)^4 \left(9810\frac{mm}{s^2}\right)(120.84mm)}{128(89.5mm)(4970mm^3)} = 0.002929\frac{mm^2}{s^2}$$

Another, simple way to solve for the viscometer constant is to simply divide the kinematic viscosity by time. This will check our answer for both volume and the constant.

$$C = \frac{v}{t} = \frac{0.7429\frac{mm^2}{s}}{253.64s} = 0.002929\frac{mm^2}{s^2}$$

Since the volume of fluid that passes through the capillary and the time it takes to do so are known, the volumetric flow rate can be calculated.

$$\dot{V} = \frac{V}{t} = \frac{4970mm^3}{253.64s} = 19.59\frac{mm^3}{s}$$

With the volumetric flow rate and the diameter of the capillary known, the average velocity of the fluid through the capillary can be found.

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\frac{\pi}{4}D^2} = \frac{19.59\frac{mm^3}{s}}{\frac{\pi}{4}(0.4600mm)^2} = 117.9\frac{mm}{s}$$

Now that the average velocity is known, the Reynolds number can be calculated to make sure flow is laminar through the capillary.

$$Re = \frac{V_{avg} \cdot D}{v} = \frac{117.9\frac{mm}{s} \cdot 0.4600mm}{0.7429\frac{mm^2}{s}} = 73$$

$$73 < 2300 \quad \therefore \quad flow\ is\ laminar$$

Since the viscometer constant is known, the kinematic viscosity of any fluid within the appropriate range for the viscometer can be found (0.6 to 3 cSt for 0C Series). The calculated value for the viscosity of water is

$$v = C \cdot t$$

$$v = 0.002929 \frac{mm^2}{s^2} \cdot 334.33s = 0.9792 \frac{mm^2}{s} = 0.9792\ cSt$$

$$v = 0.9792\ cSt$$

Next the volumetric flow rate of the water through the capillary is found.

$$\dot{V} = \frac{V}{t} = \frac{4970mm^3}{334.33s} = 14.86 \frac{mm^3}{s}$$

Using $\dot{V}$, V_{avg} is found.

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{14.86 \frac{mm^3}{s}}{\frac{\pi}{4}(0.4600mm)^2} = 89.44 \frac{mm}{s}$$

Finally the Reynolds number is calculated to verify that the flow is laminar and thus that the derivation of the viscometer equations was valid.

$$Re = \frac{V_{avg} \cdot D}{v} = \frac{89.44 \frac{mm}{s} \cdot 0.4600mm}{0.9792 \frac{mm^2}{s}} = 42.01$$

Uncertainty

Two independent sources of uncertainty are considered; uncertainty in the temperature measurement and uncertainty in the time measurement. The uncertainty in the temperature measurement is taken to be 1.8°C (1°F) and the uncertainty in the time measurement is taken to be 1.0 seconds. The resulting percent uncertainty in the kinematic viscosity is calculated using the shorthand method.

$$\%U_v = \%U_T + \%U_t$$

Where $\%U_T$ is the percent uncertainty in the temperature measurement and $\%U_t$ is the percent uncertainty in the time measurement. The calculations are as follows

$$\%U_\upsilon = \left(\frac{1.8°\mathrm{C}}{23.5°\mathrm{C}} \times 100\%\right) + \left(\frac{1s}{330s} \times 100\%\right)$$

$$\%U_\upsilon = 7.65\% + 0.33\% = 7.98\% \approx 8.0\%$$

The uncertainty present in the calculation of the Reynolds number, which is performed in order to confirm the validity of the laminar flow assumption that the equation used depends on, is a function of the uncertainty in kinematic viscosity and time (contained in the velocity value), $Re = \frac{VD}{\upsilon}$; where here V is velocity. The uncertainty was calculated using the shorthand method and was found to be

$$\%U_{Re} = \%U_\upsilon + \%U_V$$

$$\%U_V = \%U_T = 0.33\%$$

$$\%U_{Re} = 7.98\% + 0.33\% = 8.31\% \approx 8.3\%$$

The above calculated uncertainties were determined from based upon the experimental conditions. In practice, however, the viscosity would be measured while the viscometer is bathed in a temperature bath controlled to ±0.02°C (per the manufacturer’s recommendations). Table A2 can be used to determine the uncertainty of the viscosity when running the test according to the manufacturer’s recommendations. Because the calibration constant is 0.002929, the uncertainty in the kinematic viscosity measurement would be ±0.19%.

Table A2 – Uncertainty values for Ubbelohde viscometers (PSL Rheotek, Essex, England)

4. Measurement Uncertainty

Order of C $(mm^2/s)s$	Direct Flow Viscometers Ubbelohde and suspended level uncertainty (%)	Reverse Flow Viscometers uncertainty (%)
0.003	±0.19	±0.27
0.01	±0.19	±0.27
0.03	±0.20	±0.27
0.1	±0.22	±0.27
0.3	±0.23	±0.27
1.0	±0.24	±0.31
3.0	±0.28	±0.34
10.0	±0.28	±0.33
30.0 and above	±0.32	±0.36