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Project Proposal

Principle

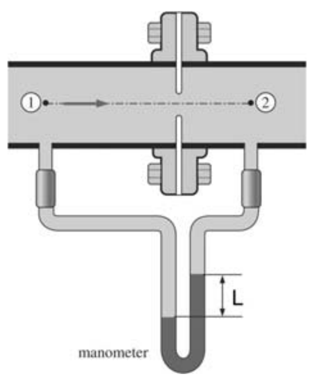
Demonstrate the Bernoulli's equation through the use of an orifice flowmeter.

Demonstration

It is proposed to build a flow measurement apparatus. The apparatus will use an orifice plate to make the measurements. The construction of the orifice plate will follow known standards and guidelines such that the discharge coefficients are known.

Given

The air flowrate from a blower is to be measured with an orifice plate as shown below. It is estimated that the blower provides 200 cfm at 70 F. If the diameter of the pipe is D = 4 in and the orifice plate diameter is d = 2 in, determine the manometer differential height, L.



Theory

Applying the energy equation between points 1 and 2

$$h_1 = h_2 + h_L \tag{1}$$

$$\frac{P_1}{\gamma_1} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_{21}} + z_2 + \frac{V_2^2}{2g} + C_d \frac{V_1^2}{2g} \tag{2}$$

where C_d is the orifice plate discharge coefficient defined as

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{Re_D^{\ 0.75}} \tag{3}$$

and

$$\beta = \frac{d}{D} \tag{4}$$

valid for $0.25 < \beta < 0.75$ and $10^4 < Re_D < 10^7$.
The mass balance between points 1 and 2 assuming constant density is

$$V_1A_1 = V_2A_2 \tag{5}$$

which can be reduced to

$$V_1 = V_2\beta^2 \tag{6}$$

Substituting into Eq. 2 and assuming horizontal flow,

$$P_1 + \frac{\rho V_1^{\ 2}}{2} = P_2 + \frac{1}{\beta^4}\frac{\rho V_1^{\ 2}}{2} + C_d\,\frac{\rho V_1^{\ 2}}{2} \tag{7}$$

a form of the Bernoulli equation. Solving for the pressure drop

$$\Delta P = \frac{\rho V_1^{\ 2}}{2}\left(\frac{1}{\beta^4} + C_d - 1\right) \tag{8}$$

This pressure drop is reflected in the differential elevation, L, in the manometer. The difference is determined by the hydrostatic equation for the manometer fluid.

$$L = \frac{\Delta P}{\gamma_m} \tag{9}$$

where γ_m is the manometer fluid.

<u>Calculations</u>

For air at 70 F and 1 atm, the density and kinematic viscosity are 0.07489 lbm/ft^3 and 1.643(10^{-4}) ft^2/s, respectively.

The Reynolds number is

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{4\dot{V}}{\pi D \nu} = \frac{4\left(200\ \mathrm{ft^3/min}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)}{\pi\left(4/12\mathrm{ft}\right)\left[1.643(10^{-4})\ \mathrm{ft^2/s}\right]} = 77{,}495$$

For $\beta = 0.5$ and the Reynolds number, the discharge coefficient is

$$C_d = 0.5959 + 0.0312(0.5)^{2.1} - 0.184(0.5)^8 + \frac{91.71(0.5)^{2.5}}{(77{,}495)^{0.75}} = 0.61$$

Substituting into Eq. 8, the pressure drop is

$$\begin{aligned}\Delta P &= \frac{\rho V_1^2}{2}\left(\frac{1}{\beta^4} + C_d - 1\right) = \frac{8\rho\dot{V}^2}{\pi^2 D^4}\left(\frac{1}{\beta^4} + C_d - 1\right) \\ &= \frac{8\left(0.07489\ \mathrm{lbm/ft^3}\right)\left(200\ \mathrm{ft^3/min}\right)^2}{\pi^2\left(4/12\ \mathrm{ft}\right)^4}\left(\frac{\mathrm{lbf\text{-}s^2}}{32.2\ \mathrm{lbm\text{-}ft}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)^2\left(\frac{1\ \mathrm{ft^2}}{144\ \mathrm{in^2}}\right)\left(\frac{1}{(0.5)^4} + 0.61 - 1\right) \\ &= 0.1839\ \mathrm{psi}\end{aligned}$$

If the manometer fluid is water, the differential level is

$$L = \frac{\Delta P}{\gamma_m} = \frac{(0.1839\ \mathrm{psi})}{62.3\ \mathrm{lbf/ft^3}}\left(\frac{144\ \mathrm{in^2}}{1\ \mathrm{ft^2}}\right) = 0.425\ \mathrm{in\ wg}$$

Using an inclined manometer at an incline of 30^o , the level is amplified to

$$L_{\mathrm{inclined}} = \frac{L}{\sin 30} = 2\ \mathrm{in\ wg}$$

Uncertainty

For a required flow measurement accuracy of ±2% (4 cfm), the manometer accuracy must be ±4% or for a measurement of 2 in wg, it must have an accuracy of ±0.08 in wg (~±1/16 in wg).