**NOTE: Thevenin & Norton Theorem**

Given a linear circuit, i.e., the terminal volt-ampere relations for the branch elements all are described by linear relations, and noting the linearity of KVL and KCL it follows that the volt-ampere relation between any two terminals must be linear. Consider the relationship between V and I at a pair of terminals connecting to the circuit as shown to the right; the linear relation has the general form \( V = R_T I + V_T \), where \( R_T \) and \( V_T \) are dimensioned constants with units of ohms and volts respectively.

The circuit on the right is a pictorial presentation of the linear expression. Thevenin's Theorem is the formal statement that, provided values for \( V_T \) and \( R_T \) are chosen properly, the two circuits will have precisely the same terminal volt-ampere relations.

There are two unknowns to determine, and two independent equations are needed for this purpose. We can for example apply an arbitrary \( V = 12 \) volts, and analyze (or measure) the circuit to determine \( I \). These provide the coordinates of one point that lies on the line relating \( V \) to \( I \). Then pick another value for \( V \), calculate \( I \) for this other value, and so locate another point on the line. That is enough information to calculate \( V_T \) and \( R_T \).

This brute force approach can be simplified by taking advantage of the fact that \( V_T \) and \( R_T \) are constants, i.e., their values are the same no matter what values for \( V \) we use to make the calculations. We simply choose particular values that simplify the calculation. There are three commonly used choices (although only two are necessary in any one circumstance). For example to calculate \( V_T \) leave the terminals open-circuit, i.e., set \( I = 0 \) and calculate the terminal voltage \( V \) under this condition. Under this condition \( V = V_T \). A second convenient choice is to short-circuit the terminals, i.e., make \( V = 0 \). Under this condition \( I = -V_T/R_T \). Assuming \( V_T \) has been calculated first, now calculate \( R_T \).

A circuit example (a) is drawn to the left. The open-circuit voltage can be calculated in several ways- a nodal analysis, a mesh analysis, or as will be used in this case by series/parallel reduction. Thus (b) shows the circuit with the \( 8 \Omega + 4 \Omega \) series resistor combination replaced by a equivalent resistor with a resistance of \( 12 \Omega \). In (c) the combination of the \( 12 \Omega \) resistor in parallel with the \( 6 \Omega \) resistor is replaced by a equivalent \( 4 \Omega \) resistor. It is the reasonably clear that the source current is \( 12/6 = 2a \).

Now work back. In (b) the \( 2a \) divides so that \( 2/3a \) flows down the \( 12 \Omega \) resistor, and \( 4/3a \) flows down the \( 6 \Omega \) resistor.

Hence in (a) the current through the \( 4 \Omega \) is \( 2/3a \) and the voltage drop across this resistor is \( 8/3v \); this is the open-circuit voltage \( V_T \).

The next calculation is for the short-circuit current; see the figure below. Since there is zero volts across the \( 4 \Omega \) resistor there is zero current through this resistor. Simplify the circuit by series-parallel combination as shown, similar to the preceding process. Determine the source current and then calculate the division of this current between the \( 6 \Omega \) and \( 8 \Omega \); the short-circuit
current is $18/19a$.

We have calculated $V_T = 8/3\text{v}$, and the short-circuit current $I = -V_T/R_T = -18/19a$. Hence $R_T = 76/27\Omega$.

The original circuit has precisely the same volt-ampere relation as the Thevenin equivalent drawn to the left. This simplification can be very useful in several circumstances. For example suppose a $5\Omega$ resistor is connected across the terminals: what is the current that flows in that resistor? This is easier to calculate from the Thevenin equivalent than from the original circuit. Of course the calculation of $V_T$ and $R_T$ represents an overhead that should be included in a comparison. But if additional questions are asked that overhead gets spread over a larger base. What is the current if the resistance is $10\Omega$, or $15\Omega$?

Here is a specific question with extensive practical application. If a resistance $R$ is connected across the terminals the power consumed by that resistor is $I^2R$. The larger $R$ is made the more the power consumed for a given current. On the other hand the larger $R$ is made the smaller the current $I$, and the smaller $R$ is made the larger the current $I$. The power dissipated always is positive, and is zero for $R=0$ and also for $R->\infty$ (I=0). Hence there is a value for $R$ for which maximum power is consumed; what is that value? The answer is determined by expressing $I$ as a function of $R$, $I = V_T/(R + R_T)$, and differentiating the expression for power with respect to $R$ to determine a maximum. The result is; set $R = R_T$. This is described as 'matching' the load $R$ to the source; you may have heard the term with respect to matching a loudspeaker to an amplifier output.

A dual relation to Thevenin's Theorem is often useful, and is known as Norton's Theorem. It amounts to rewriting the equation $V = R_T I + V_T$ as $I = (V - V_T)/R_T$ and observing that this is the terminal relation for the circuit drawn to the right. $V_T$ and $R_T$ are calculated in the same way in either case. Convenience is the principal criteria for choosing one or the other representation.

Note: It is noted above that there are three commonly used conditions to determine the Thevenin (or Norton) parameters. The two described are useful for experimental determination of an equivalent circuit in that they involve measurements that can be made (carefully) at a single terminal pair. Of occasional convenience for analytical evaluation is the following condition. Turn all circuit sources ‘off’, i.e., set voltage sources to zero (in effect making them short-circuits) and set current sources to zero (in effect making them open-circuits). The resistance seen ‘looking into’ the terminals then is the Thevenin resistance.

The calculation is illustrated for the example circuit below.

$$R_T = 4 ||([8 + (2||6)]) = 76/27 \Omega$$
Addendum

A special case involving a dependent source drawn below, more academic than particularly meaningful, was called to my attention because of a difficulty encountered in calculating the Thevenin equivalent circuit.

The short-circuit current for this circuit is zero. The open-circuit (Thevenin) voltage for this circuit is indeterminate; any value for $I$ satisfies KVL and KC. The Thevenin resistance is thus also indeterminate. Thus the equivalent circuit, either Thevenin or Norton, is indeterminate. But then the circuit itself is fundamentally indeterminate; not enough information is provided to define the terminal volt-ampere relation. In this case the circuit accommodates any terminal current that is specified. Since there is no constraint on the current through the dependent source it will be whatever is necessary to satisfy KCL, and any division of the terminal current between the resistor and the source can be accommodated. This also is the case if a terminal voltage is specified. In this case $I$ is determined, but again there is no constraint on the terminal current. In short, this circuit accommodates any terminal voltage-current pair required by what is connected to the terminals.

Note that such formal difficulties can arise even in the simplest circumstances for idealized elements. For example, what is the Norton equivalent of a lone independent voltage source of strength $V$? The open-circuit voltage is of course $V$. The short-circuit current $\rightarrow \infty$! Hence the equivalent resistance involved is $V/\infty \rightarrow 0$.

Another illustration of an indeterminate circuit connection that readily comes to mind is that drawn to the right. The current source in this circuit is redundant. KVL requires only that the voltage across the current source be $V$, the value of the current $I$ can be specified arbitrarily. Any difference between the terminal current and $I$ will flow through the voltage source. The parallel combination is electrically equivalent to the voltage source alone. For this case the short-circuit (zero resistance) current is $V/0 \rightarrow \infty$. The open-circuit voltage is $V$. The Thevenin resistance is $V/\infty \rightarrow 0$. The Thevenin equivalent circuit formally consists of a voltage source $V$ in series with a short-circuit. The Norton equivalent circuit formally consists of a current source whose strength approaches $\infty$, shunted by a short-circuit an indeterminate arrangement.!