The 'rules' of the circuit analysis game are: KVL, KCL, and the constitutive relations. ALL the answers are found therein. In this problem the parameters for the constitutive relations are not known.

The current in R3, directed towards the current source is 6 ma (KCL). Hence the current in R1, directed towards the voltage source is 2 ma (KCL). Hence Io = 8 - 2 = 6 ma.

On recognizing that the 8 kΩ and 4 kΩ resistors are in series it follows that the voltage at node 'a' is 3 Vo. (Why?) Then the current I1 is 3Vo/6k = Vo/2 ma, and so the current through the 8 kΩ and 4 kΩ resistors is Vo/4 ma. (Why?) Hence the current in the 2 kΩ directed towards node 'a' is 3Vo/4. (Why?) Then (12 - 3Vo) = (3Vo/4)(2) (This is Ohm's Law with care taken about the units involved) or Vo = 8/3v. Then I1 = (3)(8/3)/6k = 4/3 ma.

The circuit diagram is drawn in an esthetically pleasing manner, but nevertheless all the branches are electrically in parallel.

a) The resistor equivalent to all the resistors in parallel has a conductance of

\[ \frac{1}{3} + \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{10}{12} \frac{5}{3} \]

which corresponds to 6/5Ω. The net current into this equivalent resistor at the 'Y' node is 6 - 12 = -4a, and so the voltage V (relative to the datum shown) is -24/5 = -4.8v. The current in each branch follows directly.

b) Use Ohm's Law to express each resistor branch current in terms of the common voltage V, and write a KCL equation at node 'Y';

\[ \frac{V}{3} + \frac{V}{12} + \frac{V}{6} + \frac{V}{4} + 12 - 8 = 0. \]

(You should identify the origin of each term.) Solve to find V = -4.8v.

The key to the efficient analysis is the relation for current division between two parallel resistors.
Given that the current in the 3Ω resistor on the right is 2a.

a) The voltage drop across this resistor then is 6v.

b) The voltage drop across the 4Ω resistor then is 12v, and the current through this resistor is 3a.

c) Hence the current through the 6Ω is 2+3 = 1a, and the voltage drop across the 6Ω is 6v.

d) The current through the 3Ω is 3+4 = 8a, and the voltage drop across this resistor is 24v.

e) The voltage drop across the 9Ω then is 36v, and the current through this resistor is 4a.

f) Thus the current through V5 is 8+4 = 12a, and the voltage drop across the 1Ω is 12v.

g) Therefore V5 = 12 + 36 = 48v.

The power generated by the 4A source is given as 24 watts. This indicates that the voltage drop from node 4 to node 5 (V54) is 6 volts; note that the reverse polarity would mean the current source is consuming power.

From this it follows that V24 = V25 + V54 = 6V, and that the current 12 through the series combination of the 1Ω and 3Ω resistors is 3/2A.

Since I1 = 4 + 12 = 11/2 A it follows that V4 (voltage relative to reference node) = 11V. And V2 = 17V, V3 = 31/2V.

Now determine I6 = I3 + I1 = 8A, and hence that V6 = -16V. Since V5 = 5V, V51 = 5 - (-16) = 21V and I7 = 7A.

Hence I5 = 8 + 7 = 15A, and V1 = 15 + 17 = 32V.

Now calculate V8 = V6 - V1 = -16 - 32 = -48V.

A summary of the branch voltages and currents is listed to the left.
The 4kΩ and the 6kΩ resistors are in parallel, and so the current through the 4kΩ is 3/2 that through the 6kΩ, i.e., 3mA. Hence the current through the 3kΩ is 5mA. From this determine the voltage drop across the 9kΩ to be 27V, and therefore the current through the 9kΩ is 3mA. (Recognizing the current directions is left to the reader!)

The current through the 2kΩ is 5 + 3 - 4 = 4mA, and this makes it clear that the voltage drop across the 7 kΩ is 35V. Hence I_s = 4 + 5 = 9 mA.

There are several variations on a similar theme which can be used to solve this problem. For example the voltage drop across the 3kΩ resistor is 6V. Hence the current in the 12kΩ resistor (in the appropriate direction) is 1/2mA, and in the 6kΩ resistor it is 1mA. The current in the 2kΩ resistor then is 3.5 mA. Hence V_s = 2(3.5) + 6 = 13V.

Don't be misled; the diagonal 12 kΩ resistors combine to form two 6kΩ equivalent resistors in series. The 12 kΩ equivalent of this shunts the remaining 12 kΩ to provide a net 6 kΩ equivalent. The voltage across the source is then 72 volts, and the voltage at the center of the 'X' is 36 volts.

Clearly knowledge of the value of either I_o or V_A alone is sufficient to calculate all the circuit voltages and currents. Suppose we use I_o and write a KVL equation around the loop: 12 = 2I_o - 4I_o + 1I_o + 2I_o + 3I_o + 6I_o.
Hence I_o = 1.2 A. V_A = 7.2 V.

V_{GE} = 12 - 10 = 14.4 volts
V_{AE} = I_o = 1.2 volts
Since $I_0$ is the current in the 3kΩ resistor the current in the parallel 6kΩ resistor is $I_0/2$. The current in the 4kΩ resistor above is $3I_0/2$. The equivalent resistance of these three resistors is 6kΩ ($6/(3+2)$ is 2, and this is in series with 4). Hence the current in the 12kΩ resistor is $3I_0/4$, and the current in the 4kΩ resistor on the left is $9I_0/4$. Hence $6ma = 9I_0/4 + 3I_0 + 3I_0/2 + 3I_0/4$, and so $I_0 = 0.8 ma$. The power absorbed in the 12kΩ resistor is $(12kΩ)(3I_0/4)(3I_0/4) = 4.32 mW$.

Observe that the current through the 3 Ω resistor is $V_0/(3 + 4|12) = V_0/6$, and that the current $I_X$ is equal to $3/4$ of this value, i.e., $V_0/8$. (This from series-parallel combinations of the split branch.) Then apply KCL at the $V_0$ node $6 = V_0/(1/4 + 1/6 + 1/12 + 1/2)$, where the '2' comes from the controlled current source contribution to the node current. Hence $V_0 = 6V$.

Since $V_0 = 2(4I_X) = 8I_X$, a similar calculation could be made using $I_X$ as the unknown.

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Extra: Apply superposition, and by inspection plus some scratch arithmetic analyse the circuit.
The easy way to complete a nodal analysis is to convert the 6 V source in series with the 2 Ω resistor to a 3 A source into node V1, in parallel with a 2 Ω resistor; do this by inspection.

Superposition works nicely here also. By inspection and a bit of scratch arithmetic obtain the "partial" voltage of V1 due to the 6 V source as 2.25 V, the partial contribution of the 12 A source as -9 V, and the partial contribution of the 4 A source as 2 V; therefore \( V_1 = -4.75 \text{ V} \).

\[
\begin{bmatrix}
3 - 12 \\
4
\end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 1 \\ -1 & 1 + 0.5 \end{bmatrix} \begin{bmatrix}
V_1 \\
V_0
\end{bmatrix}
\]

\( V_1 = -4.75 \text{ V} \)
\( V_0 = -0.5 \text{ V} \)

\[
\begin{bmatrix}
-9 \\
4
\end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1.5 \end{bmatrix} \begin{bmatrix}
V_1 \\
V_0
\end{bmatrix}
\]

\( I_0 = V_0 / 2 = -0.25 \text{ A} \)
\( I_1 = 12 - 4 + 10 = 31 / 4 \text{ A} \)

Nodes 1 and 2 are used as a 'supernode' to apply KCL without having to use the current in the 6 volt source branch. A second equation is provided by the node voltage difference constraint.

\[
\frac{V_1 - 3}{6} + \frac{V_1}{12} + \frac{V_2}{6} + \frac{V_2 + 3}{12} = 0
\]

\( V_2 - V_1 = 6 \)

Note: resistance in kΩ, current in mA
The novelty in this problem is the appearance of independent voltage source branches not in series with a resistor. Ordinarily something such as supermode equation would be needed, but this problem is less imposing than might appear at first sight. By inspection (and KVL) \( V_1 = 12 \) \( V \) and \( V_3 = 6 \). There is then just one unknown and the \( V_2 \) node is just the place to apply KVL.

This is what is done in a more formal manner below. Also another point of view also is described; more work is traded for less thinking.

Note: A somewhat different view of the supernode calculation, perhaps easier to follow but at a price of more variables, is to define a current variable for each independent voltage source branch. For each such branch an independent KVL equation can be written relating the branch voltage difference to the source strength. Thus as many independent equations can be written as the number of new variables introduced, and the enlarged set of equations can be solved. The enlargement is the price for the procedural simplification. This is not too stiff a price if a computer does the numerical computations.

\[
\begin{bmatrix}
12 \\
6 \\
0
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & -0.5 & 0.5 + 0.5 + 1 & -0.5 \\
-6 & -1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\]

\[
\begin{align*}
V_1 &= 12 \, V \\
V_2 &= 4.5 \, V \\
V_3 &= 6 \, V
\end{align*}
\]

\[
I_0 = 4.5 \, A
\]

Note: The supernode used is enclosed by the dashed rectangle, and the corresponding KCL equation is written in place of the equation that would be written at node \( V_1 \). Do not forget to allow for all branches exiting the supernode. The KCL equation at node \( V_2 \) is replaced by a KVL equation.

\[
\begin{bmatrix}
4 \\
12 \\
0
\end{bmatrix} =
\begin{bmatrix}
1+1 & 1+1 & -1-1 \\
-1 & 1 & 0 \\
-1 & -1 & 1+1+1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\]

\[
\begin{align*}
V_1 &= 7.5 \, V \\
V_2 &= -4.5 \, V \\
V_3 &= 1 \, V
\end{align*}
\]
### Problem 3.22

The supernode KCL equation is:

\[ 0 = (V_2 - 6)/1 + V_2/1 + V_1/2 + [I_x = -(V_2 - 6)/1] \]

and the KVL equation is \( V_1 - V_2 = 12 \)

Solve to get \( V_1 = 8V, V_2 = -4V, Y_0 = 4V \).
Problem 3.34

The KCL equation for the supernode is:

\[ \frac{V_1}{2} + \frac{V_1}{2} + \frac{V_2}{4} = 4 \text{ or } 4V_1 + V_2 = 16 \]

and the KVL expression is \( V_1 - V_2 = 4\left(\frac{V_1}{2}\right) \)

or \( V_1 = -V_2 \)

Solve for \( V_1 = 16/3V \) and \( V_2 = -16/3V \)

Problem 3.43

Only one mesh equation is needed; the independent current sources determine the other mesh currents by inspection. Thus

\[ (2 + 4 + 2)I_1 = 12 + (4)(2) + (-2)(2) \]

or \( I_1 = 2A \)

Hence \( V_0 = 8V \)

Problem 3.46

\[-2 + \frac{V_2}{2} + \frac{V_1 - V_3}{4} = 0 \; ; \; \text{KCL @ node 1} \]

\[-\frac{V_2}{2} + \frac{V_2 - V_3}{2} + \frac{V_2}{6} = 0 \; ; \; \text{KCL @ node 2} \]

\[\frac{V_3}{6} + \frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{2} = 0 \; ; \; \text{KCL @ node 3} \]

Note: resistance in k\(\Omega\), current in mA
Problem 3.50

\[ \begin{align*}
\begin{vmatrix} 24 \end{vmatrix} &= \begin{vmatrix} 2 + 2 + 2 & -2 - 2 & 1 + 3 + 2 \end{vmatrix} \\
&= \begin{vmatrix} 1 \end{vmatrix} \\
&= \begin{vmatrix} 2 \end{vmatrix} \\
&= 24 \quad l_0 = 24/7 \\
&= l_1 = 36/7 \\
&= l_2 = 24/7 \\
&= V_0 = 72/7
\end{align*} \]

Method #1
Instead of two mesh equations write one supermesh equation excluding the current source branch: \( 24 = 2l_1 + (1 + 3)l_2 \) and use the constitutive relation \( l_2 = \frac{l_1}{2} \) or \( l_2 = \frac{5}{11} l_1 \).

Method #2
Alternatively include the voltage drop across the dependent source as an added variable, and add the constitutive relation as an additional independent equation:

\[ \begin{align*}
\begin{vmatrix} 24 \end{vmatrix} &= \begin{vmatrix} 2 & 0 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 1 \end{vmatrix} \\
&= \begin{vmatrix} l_1 \end{vmatrix} \\
&= \begin{vmatrix} l_2 \end{vmatrix} \\
&= \begin{vmatrix} V_x \end{vmatrix}
\end{align*} \]

In either case solve to find:

\[ l_1 = \frac{12}{11} \quad V_x = \frac{240}{11} \]
\[ l_2 = \frac{60}{11} \quad V_0 = \frac{180}{11} \]
**Problem 3.54**

![Circuit Diagram]

Method #1
Use the current source constitutive relation \( i_3 = -6A \) as an independent equation to replace the \( i_3 \) loop equation.

\[
\begin{bmatrix}
-3 & 4 & 0 & 0 & 0 & 11 \\
3 & 0 & 12 & -12 & 0 & 12 \\
-6 & 0 & 0 & 1 & 0 & 13 \\
0 & -4 & 0 & -3 & 3 & 14 \\
\end{bmatrix}
\]

Replace loop \( i_3 \)

Method #2
Define the voltage drop across the current source as a new variable, and add the constitutive relation as an additional independent equation.

\[
\begin{bmatrix}
-3 & 4 & 0 & 0 & 0 & 0 & 11 \\
3 & 0 & 12 & -12 & 0 & 0 & 12 \\
0 & 0 & -12 & 12+3 & -3 & 1 & 13 \\
0 & -4 & 0 & -3 & 3 & 0 & 14 \\
-6 & 0 & 0 & 1 & 0 & 0 & \bar{V}_x \\
\end{bmatrix}
\]

Method #3
By inspection note that \( i_1 = -3/4A \). Hence the dependent source voltage is \( 3V \), and so the voltage drop across the \( 6A \) source is \( \bar{V}_x = 0 \). \( i_3 = -6A \) of course. Since \( 3*(i_3-i_4) = 3 \) find \( i_4 = -7A \).
Similarly \( 12*(i_2-i_3) = 3 \) and so \( i_2 = 23/4A \). These are the same results, of course, as would be obtained by solving the mesh equations above.

**Problem 3.57**

![Circuit Diagram]

\[
\begin{vmatrix}
-12 \\
0 \\
\end{vmatrix} = \begin{vmatrix}
4 & 4 & +8 & 0 \\
-8 & 2 & +8 & \end{vmatrix} \begin{vmatrix}
1_1 \\
1_2 \\
\end{vmatrix}
\]

- \( i_1 = 3/4A \)
- \( i_2 = 3/5A \)
- \( V_{out} = 24/5V \)
Problem 3.64

Voltage here is $V_1 + 6V$

$$\begin{vmatrix}
-1 & -3 \\
-2 & -1/2 \\
2 & 0 \\
4 & 3
\end{vmatrix} = \left[ \begin{array}{cccc}
-1/2 & -1/2 & 0 & -1/2 \\
-1/2 & 1/2 + 1/2 & 0 & 0 \\
0 & 0 & 1/3 + 1 & -1 \\
-1/2 & 0 & -1 & 1/2 + 1/2
\end{array} \right] \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \end{Bmatrix}$$

$$v_1 = 2V$$
$$v_2 = -2/3V$$
$$v_3 = 11V$$
$$v_4 = 38/5V$$

Here (below) is a solution in which Superposition is used.

Here is a narrow solution, obtained by first finding the Thevenin equivalent circuit (using Superposition) seen by the series combination of the 6V source and the 2Ω resistor. The Thevenin circuit consists of a 27V source in series with a 7Ω resistor, etc.
Problem 3.65

\[ \begin{bmatrix} 0 \\ -1 \\ -\frac{1}{4} \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + 1 \\ -1 \\ \frac{1+1}{1+2} \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1+1 \\ -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} \]

\[ V_1 = 14.844 \, V \]
\[ V_2 = 11.5 \, V \]
\[ V_3 = 12.906 \, V \]
\[ V_4 = 11.156 \, V \]
\[ V_5 = 10.312 \, V \]
\[ V_0 = \frac{V_5}{3} = 3.437 \, V \]

Problem 3.72

\[ V_{\text{out}} = \left( \frac{R_2}{R_1 + R_2} \right) \left( v_1 - v_2 \right) \left( -\frac{R_4}{R_3} \right) \]
Problem 3.74

Writing node equations here is awkward but...

\[
\begin{array}{cccc}
 v_1 & 1 & 0 & 0 \\
 0 & -1/R_1 & 1/R_1 + 1/R_2 & -1/R_2 \\
 0 & 0 & -1/R_2 & 1/R_2 + 1/R_3 + 1/R_4 \\
 0 & 0 & 1 & 0 \\
\end{array}
\]

Write directly:

\[
\begin{align*}
l_1 &= l_2 = \frac{v_1}{R_1} \\
v_2 &= v_1 - l_2 (R_1 + R_2) = -v_1 \frac{R_2}{R_1} \\
l_3 &= \frac{v_2}{R_3} = -v_1 \frac{R_2}{R_1 R_3} \\
l_4 &= l_2 - l_3 = \frac{v_1}{R_1} \left(1 + \frac{R_2}{R_3}\right) \\
v_{\text{out}} &= v_2 - l_4 R_4 \\
&= -v_1 \frac{R_1}{R_1} \left(\frac{R_2}{R_1} + R_4 \left(1 + \frac{R_2}{R_3}\right)\right)
\end{align*}
\]
By inspection (resistance reduction, etc) partial $V_o$ associated with the voltage source is

$$\frac{12}{3 + 6|| (8 + 2)} \cdot \frac{6}{6 + (8 + 2)} \cdot 8 = \frac{16}{3} \, \text{V}$$

current in $3\,\Omega$ current resistance divider

current in $6\,\Omega = \frac{2}{2 + (6 + (3||6))} \cdot 2$

and partial $V_o = 8/3 \, \text{V}$

$$V_o = \frac{8}{3} + \frac{16}{3} = \frac{24}{3} \, \text{V}$$
Solve (just for fun) using node equations. Note that $2000l_x = v_1$, and so $v_2 = 2v_1$.

$$3 = \frac{v_1 + 6}{6} + \frac{v_1 + 2v_1}{8} \quad \text{and} \quad v_1 = \frac{24}{11} = v_0$$

And using Thevenin's theorem:

**Thevenin Resistance**

$$i = \frac{3}{2}l_x$$

$$v = 4i + 2l_x + 2l_x$$

$$\frac{v}{i} = 7 \, \text{k} \Omega$$

**Open-Circuit Voltage**

$$2l_x = 6(3-l_x) - 6 \quad \text{and} \quad l_x = \frac{3}{2} \, \text{ma}$$

$$v_T = 2l_x + 2l_x = 6v$$

$$\gamma_0 = 6 \cdot \frac{4}{7 + 4} = \frac{24}{11} \, v$$
For the Norton (= Thevenin) resistance turn off all sources and calculate resistance looking back into the circuit from the 2kΩ resistor. This is 6kΩ (3kΩ+3kΩ) = 3kΩ. The Norton (short-circuit) current (directed in the same direction as the Io arrow) is readily calculated using superposition. The contribution of the current source is 1mA (half the source current), and the contribution from the voltage source also is 1 mA (6V/6kΩ). Hence the short-circuit current is 2mA. Then the current through the 2mA is 2(3/2+3) = 1.2 mA.

For maximum power transfer set RL = RT.

Load Power = $\left(\frac{8k}{4k+4k}\right)^2 \cdot 4k = 4\text{mW}$
\[ I = -(100+1)I \]
\[ V = -I(1k+9k) \]
\[ R_L = R_T = \frac{V}{I} = \frac{1k+9k}{100+1} = 99\Omega \]