INTRODUCTION TO OPERATIONAL AMPLIFIERS

Amplifiers
For many purposes a simplified approximation for the characteristics of a basically nonlinear electronic device may be used to provide very useful analytical information and understanding about the device performance without the computational burdens of nonlinearity and fine structure. The operational amplifier is an important electronic device for which this sort of useful simplification will be illustrated. The operational amplifier is typically a voltage amplifier often operated under conditions to assure certain approximations are valid. It turns out that this constraint on operation still encompasses an extraordinarily diverse number of important applications. The discussion here is limited largely to circumstances where this constraint is applicable.

A linear 'voltage amplifier' may be described fundamentally as an electronic device which uses an internal energy source to produce an output voltage \( V_o \) that is a magnified copy of a voltage \( V_i \) between its inputs terminals. The voltage 'gain' is the magnification provided, and the device is linear because the magnification corresponds mathematically to a multiplication by a constant.

In its practical realization a voltage amplifier has a voltage transfer characteristic generally as illustrated by the transfer characteristic to the left in the figure below; a conventional icon representing a voltage amplifier is drawn at the top of the figure. (The negative output terminal often is implied rather than shown explicitly.) Note that in the figure there are two voltage polarity markings at the input. One refers just to the description of the input voltage \( V_i \); that voltage is determined by the circuit in which the amplifier is imbedded. The other refers to the relationship between the polarity of the voltage applied to the input and the voltage at the output, and is an intrinsic property of the amplifier circuit itself.

An input voltage drop with respect to the amplifier input \( \pm \) symbols produces a voltage drop as described by the output polarity markings. This is an intrinsic characteristic of the amplifier design that the manufacturer is describing. Clearly the actual input voltage could be either a voltage rise or a voltage drop; all the specification indicates is the relative polarity of the input and the output. The manufacturer does not seek to specify how you should describe the voltage you apply. Normally however the convenient thing to do, and it is more or less universally done, is to use the amplifier polarity markings for both purposes.

A piecewise linear approximation to this characteristic (shown by the middle graph) captures certain essential features of this voltage transfer relationship in a mathematically simplified form. The amplifier has a range of output voltages intrinsically limited by the properties of the components with which it is assembled. The limits are set largely by the voltage supply levels used to power the amplifier, and the amplifier is said to be 'saturated' when it reaches one or the other limit.

Between output voltage limits of \( \pm V_{sat} \) the amplifier exhibits an approximately constant voltage gain \( G \gg 1 \). This piecewise linear approximation often is quite useful for making design estimates, and for appreciating the relative importance of different circuit parameters on overall circuit performance. It can provide a quantitative basis for evaluating whether a circuit design is sufficiently mature to warrant a more detailed computer numerical analysis; often enough it is sufficiently accurate in itself to be quite useful.

We go one step further in modeling the amplifier transfer characteristic. An idealized voltage amplifier is a conceptually abstract electronic device for which \( G \to \infty \), i.e., the input voltage range over which the gain is obtained becomes very small compared to the saturation voltage magnitude. This is depicted by the
characteristic drawn on the right of the figure. It is important to appreciate that $G \approaches \infty$, rather, as is too often loosely stated, $G = \infty$. Keeping this distinction in mind helps to avoid conceptual problems that can arise, often subtly, where mathematically singular circumstances are involved. Thus if the voltage gain is large rather than infinite a finite output voltage is produced by a very small input voltage; if the gain is large enough the input voltage could well be negligibly small compared to other circuit voltages. If the gain approaches $\infty$ the idealized device input voltage approaches zero, i.e., becomes negligibly small (as long as the output voltage is within the saturation voltage limits). It is much easier to conceptualize the possibility of a negligibly small but nonzero input producing a very large but finite amplifier output than puzzling over how a zero input voltage can do so. (Or worse, for example, how a zero input voltage produces different finite output voltages.)

Incidentally it might be inferred (correctly) that for the singular idealization $G \rightarrow \infty$ to be meaningful the amplifier must be imbedded in additional circuitry which somehow enables the amplifier output voltage to be evaluated without explicit knowledge of a neglected input voltage. That is, the input voltage difference may be negligible in comparison to other individual voltages at the amplifier terminals but it is not unimportant; it is that negligibly small difference that drives the amplifier. It is necessary to determine the measurable effect of this 'negligible' voltage by indirect means, and by implication there must be additional circuitry connecting the amplifier inputs and output which enable this determination. The technical jargon is that there must be 'feedback' between inputs and output. Example circuits illustrating how the output voltage may be calculated are presented later.

There is one more important presumption made for the amplifier idealization which we simply take note of here. An electrical signal applied at the amplifier input does not appear instantaneously at the amplifier output; there is a finite ‘propagation time’ between input and output. This delay ordinarily is quite small, in particular in the propagation time interval the input signal does not change significantly. In such circumstances the current output signal may be considered to be due effectively to the current input signal. This is an important consideration in connection with ‘feedback’ from the output to the input.

**OpAmp**

In many useful applications, a surprisingly large number of applications in fact, actual amplifier performance is closely approximated by an idealized amplifier model. Indeed not infrequently circuits are designed explicitly to insure acceptability of this approximation. And in other cases where the idealization is not a sufficiently accurate approximation nevertheless it often provides a starting point for an iterative process towards a final design. Consider the 741 amplifier, an older but proven industry-standard device, which has a voltage gain exceeding $10^5$ in normal operation. To cause an output voltage change between representative saturation voltage limits of ±15 volts, i.e., a full thirty-volt output change, the input voltage change involved is less than 0.3 millivolt. Such a small voltage difference often may be neglected, i.e., approximated as zero, when compared to other circuit voltages with which it is associated in a KVL loop equation.

Another characteristic of an idealized amplifier is that of 'infinite' input resistance, i.e., there is no current flow into or out of the amplifier-input terminals. The practical approximation to this, and a more meaningful interpretation of what is realized in practice, is that the amplifier input resistance is so large that for the small input voltage differences associated with a high gain amplifier the input current is negligible compared to other circuit currents. The idealization to an infinite input resistance simply is a mathematical formalism corresponding to neglecting amplifier-input currents compared to other circuit currents in a KCL node equation. The 741 amplifier has a nominal input resistance of 50kΩ, corresponding to terminal currents no more than 6 nanoampere over the gain region of operation.

The final idealization involved in these experiments concerns the resistance seen looking back into the amplifier output; the idealization is a negligibly small ($\rightarrow$ zero) output resistance. This means that the amplifier circuit looking back into the output is essentially just a voltage source with no series resistor (negligible Thevenin resistance). The 741 is designed for a nominal output resistance of 100Ω.

An amplifier with the general characteristics of very high voltage gain, very high input resistance, and very low output resistance generally is referred to as an OpAmp, a colloquial contraction of 'operational amplifier'. This description is historically based; such amplifiers were first used to perform various
arithmetic and related operations associated with analog computers. They have proved to be extraordinarily useful in many other applications as well, so useful that their usage has increased despite the decreased use of analog computer.

Some additional common nomenclature: The + input terminal is called the 'noninverting' input; an input voltage applied with a polarity as indicated by the terminal markings produces an in-phase output voltage. The - terminal is the 'inverting' input terminal.

**Non-Inverting Voltage Amplifier**

The OpAmp (assumed to be the idealized device in this note) in the circuit shown to the right is used in conjunction with a resistive voltage divider to force a fixed relationship between the input and output voltages. A circuit input voltage $V_i$ (distinguished from amplifier input terminal voltage difference) is applied between the noninverting (+) input and ground; there is no (more precisely, negligible) current into the input because of the infinite (more precisely, high) input resistance of the OpAmp.

Because of the high amplifier voltage gain, the voltage difference between the amplifier (not the circuit) inputs is negligibly small compared to other circuit voltages. Because the voltage difference is negligible the voltage at the inverting input is essentially $V_i$, the same as at the noninverting input.

A particularly significant aspect of the 'equal' voltages at the two input terminals is that the equality is produced without an electrical current flow between the inputs, quite different in this respect from the use of a short-circuit connection between the inputs to obtain voltage equality. Because there is no current into the amplifier terminals $R_1$ and $R_2$ carry the same current and so are electrically in series. The voltage divider action by these resistors forces a specific relationship between the amplifier output voltage and the inverting input voltage, and therefore so also with the essentially equal noninverting input voltage;

$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

Note that the negligible (although nonzero) value of the amplifier input voltage is not used to evaluate the output voltage, only the presumption that it is negligible compared to the voltage across $R_1$ is used. The actual small but finite value will be whatever is required to satisfy fundamental circuit laws; in this case that requirement is expressed via the voltage divider. The resistive divider provides the 'feedback' from the output to the input necessary to evaluate the circuit behavior.

Since an input voltage of less than a millivolt or so produces output voltages of the order of ±15 volts (representative saturation limit) for the 741 the idealized analysis should be reasonably valid for a voltage across $R_1$ (negligibly different from the circuit input voltage) greater than, say, 100 millivolt or more. Hence this OpAmp circuit can be used to realize a voltage gain up to roughly $15/0.1 \approx 150$ or so.

One might well ask after the rationale for taking an amplifier with an intrinsic gain of $10^5$ and combining it with other circuitry to get a voltage gain orders of magnitude smaller. What makes this tradeoff very acceptable lies with the fact that the gain so obtained depends only on the ratio of two resistances. That ratio can be reproduced from one amplifier assembly to another to within fine tolerances and moreover maintained over a wide range of environmental changes. This is far better than the gain stability of the OpAmp alone (the intrinsic gain may vary by a factor of three or more from one device to another). This assured dependability and reproducibility is one justification for accepting the lower gain. This assurance is complemented by the predictability of the voltage gain.

A special case of the noninverting amplifier of some considerable importance is that for which $R_1 \to 0$ and $R_2 \to \infty$, as illustrated to the right. This configuration is called a 'voltage follower'. The inverting input voltage is the same as the output voltage, and as long as the amplifier maintains a very high gain (i.e., is not saturated) it forces the inverting input to track the input voltage. The special advantage offered by this amplifier
configuration is that because of the high input resistance of the opamp it provides at its output a copy of $V_i$ with minimum effect on the circuit in which $V_i$ is measured.

**Add-On**

To understand more thoroughly how the amplifier output is established it is important to recognize that the amplifier input terminal voltage difference is very small but not zero, and the amplifier gain is very large but not infinite. Thus suppose for some reason whose details we need not be concerned with the amplifier output decreases momentarily. The voltage fed back to the inverting input therefore is reduced. But this means the (small but finite) amplifier input voltage increases and the large (but finite) voltage gain causes the output voltage to increase. The feedback connection here provides a mechanism that mitigates the decrease in output voltage. Similarly if the output voltage increases the feedback effect reduces the input voltage and so the output voltage. Although the amplifier input voltage may be negligibly small compared to other voltages it is essential to operation of the amplifier that it is finite.

**Trans-resistance amplifier**

Another deceptively simple OpAmp circuit, the 'transresistance' amplifier circuit, is drawn to the right. Transresistance is a contraction of the words 'transfer resistance', and refers to the ratio of a voltage across one pair of terminals to a current associated with a different terminal pair. In the present case the ratio is between the amplifier output voltage $V_o$ (relative to ground) and the input current $I$. Note that the noninverting input is connected to ground, and the input current (from a source not shown) feeds the inverting input. Assume that (normal) circuit operation in this experiment will be such as to maintain the amplifier output voltage between the saturation limits, and that the idealized OpAmp approximations are valid. The resistive feedback connection between output and input should be noted, i.e., this is the mechanism that ties the amplifier output to conditions at the input involving variables whose values are being neglected.

Because the input voltage difference to the amplifier will be zero (more precisely, negligibly small) the inverting input is maintained at ground potential. The inverting input under this condition is said to be a 'virtual' ground, i.e., maintained at ground potential because of the high gain of the OpAmp but not connected physically to ground. A KCL equation written at the inverting input, and assuming the current into the amplifier is negligible compared to $I$, indicates that $I$ flows into $R$. Then assuming negligible amplifier terminal voltage difference it follows that $V_o = -RI$.

To provide additional insight into the nature of the idealized OpAmp approximation a somewhat more detailed (i.e., less idealized) model is used to analyze the circuit behavior of the transresistance amplifier, as illustrated in the figure to the right. The extended (for the present illustration only) amplifier model assumes a finite constant voltage gain $G$ (assuming unsaturated operation) and a finite input resistance $R_i$. The amplifier 'holds' the relationship between the voltage difference across its input terminals and the output voltage as shown in the figure; note the polarity relationship between input and output voltages. (If the gain $G$ were 'infinite' the input voltage difference producing the finite output voltage $V_o$ would be 'zero'.) Applying KCL at the inverting input terminal provides the equation

$$ I = -\frac{V_o}{R} \left[ 1 + \frac{1}{G} + \frac{R}{R_i} \right] $$

The idealized amplifier approximations are $G \gg 1$ (and so $1 \gg 1/G$) and $GR_i \gg R$ (the latter condition corresponds to asserting that the current through $R_i$ is much smaller than the current through $R$). Or saying the same thing somewhat differently, these mathematical approximations correspond physically to assuming the inverting input is a virtual ground (zero input voltage difference) and neglecting amplifier input current (infinite input resistance). On applying these approximations the transresistance expression simplifies to the same result obtained on applying the idealized model conditions directly, i.e., $V_o = -IR$. 

---

Circuits  OpAmp  4  M H Miller
Inverting Voltage Amplifier
The circuit drawn to the right takes advantage of idealized transresistance amplifier performance to provide a well-defined inverting voltage gain, similar to that of the non-inverting configuration described before. Because the idealized amplifier inverting input is a virtual ground in this circuit the input current I is

\[ I = \frac{V_i}{R_i} \]

Hence

\[ V_o = -IR_o = -\frac{V_iR_o}{R_i} \]

The voltage gain \( V_o/V_i \) depends on comparatively stable resistance values, and not on the much more variable amplifier characteristics. Moreover the gain depends on relative resistance values, not an absolute resistances. Integrated circuit processes, for example, produce resistors with fairly large variation about a nominal value. However process variations affect resistors produced concurrently a few thousandths of an inch apart more or less in the same way, and resistance ratios are quite reproducible. Amplifier voltage gain determined by a resistance ratio thus can be manufactured with relatively high precision and stability.

'Adder' Circuit
The general idea of the inverting amplifier circuit can be extended, as illustrated in the circuit diagram to the right, to provide an output voltage that is the weighted sum of two input voltages. The virtual ground property of the inverting node in this circuit, and the KCL requirement at the same node provides the upper of the two equations shown just below the circuit drawing. The second equation simply is an algebraic re-expression of the first equation that emphasizes the weighted-sum property. A special case of some interest is the equal-weight sum obtained for \( R_1 = R_2 \); the output then is proportional to the sum of the inputs.

\[
\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_o}{R_o} = 0
\]

\[
V_o = -R_o \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} \right)
\]
OPAMP PROBLEMS (and answers)

1) Assume an idealized voltage amplifier and show that the transresistance V/I is -(N+2)R where N is a constant.

**Answer** The only novelty added in solving idealized opamp problems is the special properties of the opamp circuit element. The 'rules' are straightforward. Neglect the voltage drop across the amplifier input terminals compared to any circuit voltages, and neglect the current into the amplifier input terminals compared to any circuit currents. The mathematical equivalent of this neglect is to set the amplifier input voltage and the input currents to zero.

Any technique of circuit analysis may be applied, some perhaps more easily than others in particular circumstances. In the present case there is no current into the inverting (-) input, and the voltage at the noninverting (+) input is the same as that at the inverting input. Hence the voltage at the common node of the three resistors is -IR. Similarly there is a current into the node from the R/n resistor of IR/(NR/N) = NI, and so the current into the amplifier output is (N+1)I. Hence V = -(N+1)I - IR = N+2)IR. The transresistance is V/I = -(N+2)R.

2) Show that the voltage gain V_o/V_i for the circuit shown is 5.

**Answer** The inverting input voltage is V_i, the same as for the non-inverting input (neglecting the amplifier input voltage). Since there is no current into the inverting input the voltage at the middle node on the right is 2V_i. Hence the amplifier output current is V_i/R + 2V_i/R = 3V_i/R, and V_o = 3V_i + 2V_i = 5V_i.

3) Show that the voltage gain V_o/V_i is (1 + (R_2/R_1))(-R_4/R_3) for the circuit shown.

**Answer** The voltage gain of the first stage is 1+(R_2/R_1), and the voltage gain of the second stage is -R_4/R_3. The cascade gain is the product.

4) Show that the current ratio I_1/I_2 is found from I_1R_1 = I_2R_2 for the circuit shown.

**Answer** Apply KVL, neglecting the voltage drop across the amplifier input I_1R_1 = I_2 R_2
5) a) Using idealized OpAmps design an amplifier with a voltage gain of -10 and an input resistance of 1 kΩ.
b) Using idealized OpAmps design an amplifier with a voltage gain of +10 and an input resistance of 1 kΩ.
c) Using idealized OpAmps design an amplifier with a voltage gain of -10 and an infinite input resistance.
d) Using idealized OpAmps design an amplifier with a voltage gain of +10 and zero input resistance.

Answer

![Diagram](image)

6) Assume idealized opamps in the circuit to the right; calculate the voltage gain V0/Vi.

Answer    Replace the two cascade amplifier stages by an equivalent single-stage amplifier with a voltage gain of -110 and an input resistance of 1KΩ. Simply to provide an estimate consider this to be an idealized opamp. Then the feedback arrangement within which it is imbedded provides an ‘idealized’ gain of -10. Of course the single-stage equivalent is not an idealized opamp. Hence the gain magnitude will be ‘somewhat less’ than 10.

![Diagram](image)