Amplifier Frequency Response

Objective
Active devices (in particular) change state in response to terminal stimuli via an internal charge rearrangement. Since charge transport is not instantaneous the rearrangement takes some time. The relationship between stimulus and consequent charge distribution may become corrupted if the stimulus is changed before an internal steady state is reached for the original stimulus. For this reason all devices have an ultimate limit to the rate of signal change for a meaningful response. This limit may be investigated either in the time domain (e.g., step function response) or in the frequency domain (sinusoidal frequency response). In this note it is frequency response that is considered.

Introduction
The essential purpose of an amplifier is to accept an input signal and provide an enhanced copy of that signal as an output. However there is a fundamental relationship between signal frequency and gain such that a given gain cannot be maintained over an arbitrarily large frequency range. Physically it takes time for electric charge in a device to redistribute itself in response to a control signal, and so the response of a device to a control signal inevitably becomes jumbled for very fast signal changes. This is an ultimate limit to circuit response; degradation of the response may begin at lower signal frequencies because of delays associated with other circuit components. Circuit components can introduce degradation of the frequency response of a circuit at low frequencies as well as high, as will be seen.

In general the frequency response of an electronic circuit, e.g., the transfer gain of the circuit, has the general appearance illustrated. There is a ‘mid-band’ range of operation for which the gain is substantially independent of frequency, bounded by ‘high’ and ‘low’ frequency ranges in which the gain is degraded. An amplifier is ‘wide-band’ if the ratio of a frequency measuring the onset of the high-frequency degradation to a corresponding frequency for low frequency degradation is relatively large. A basic audio amplifier, for example, has a substantially ‘flat’ response extending from about 100 Hz to roughly 10KHz. ‘Narrow-band’ amplifiers, used for more specialized purposes, approximate selective amplification at a single frequency. Our basic interest here is in wide-band amplifiers.

To simplify consideration of the frequency response of wide-band amplifiers analysis generally is separated into three frequency ranges. The argument used to justify this separation is that those circuit components associated with low-frequency degradation have by definition lost significant influence on the response in mid-band, and supposing a monotonic behavior have no influence on the high-frequency response. The converse argument removes the influence at low frequencies of those components affecting the high frequency response. And, of course, in mid-band by definition neither of these sets of components influences the response significantly. The mid-band range is the one assumed in previous work, for example by neglecting the influence of coupling and bypass capacitors.

Initially we assume that frequency constraints are associated with circuit components other than the active devices. Taking intrinsic device limitations into account is done later as an extension of the basic procedures.

Poles, Zeros, and the Half-Power Frequency
The transfer function of a linear circuit has the general form
\[
\frac{G_H}{G_{MB}} = \prod \left(1 + \frac{j\omega}{\omega_p}\right) \prod \left(1 + \frac{j\omega}{\omega_z}\right)
\]
in terms of the circuit poles and zeros, $\omega z_\alpha$ and $\omega p_\beta$ respectively. $G_{MB}$ is the frequency-independent mid-band gain; the high-frequency gain $G_H \to G_{MB}$ as $\omega \to 0$, i.e., as the frequency decreases out of the high-frequency range. A series expansion then provides as a first-order estimate of the response

$$\frac{G_H}{G_{MB}} = 1 + j \omega \sum_{\alpha, \beta} \left( \frac{1}{\omega z_\alpha} - \frac{1}{\omega p_\beta} \right) + \ldots$$

The first-order approximation approximates the actual response using a single fictitious pole; this pole provides the most common measure used to locate the onset of high-frequency degradation. At the pole the circuit gain is degraded from the mid-band response by a factor $\sqrt{2}$, i.e., 3 dB:

$$\omega_{3db} = \frac{1}{\sum_{\alpha, \beta} \left( \frac{1}{\omega z_\alpha} - \frac{1}{\omega p_\beta} \right)}$$

A similar series expansion may be made for the low-frequency response, this time in terms of $1/j\omega$ since the mid-band gain is approached as frequency is increased out of the low-frequency range. Thus

$$\frac{G_L}{G_{MB}} = \prod_{\omega} \frac{(1 + j \omega z_\alpha)}{\omega} \prod_{\beta} \frac{(1 + j \omega p_\beta)}{\omega} = 1 + \frac{1}{j \omega} \sum_{\alpha, \beta} (\omega z_\alpha - \omega p_\beta) + \ldots$$

and

$$\omega_{3db} = \sum_{\alpha, \beta} (\omega p_\beta - \omega z_\alpha)$$

The expressions derived presuppose knowledge of the circuit poles and zeros. However a circuit analysis program could compute the frequency response directly more easily than by first computing poles and zeros. The equations ordinarily are more useful as approximations for circuit synthesis, i.e., circuit elements are to be specified so as to provide a specified frequency response.

**CE Amplifier Illustration**

The single-stage CE BJT amplifier circuit shown to the right is used to illustrate the frequency response approximate calculation. First observe that the 0.1 µF input and output coupling capacitors clearly affect the low-frequency response. As signal frequency decreases, and the capacitative reactance increases, the fraction of the source voltage transferred to the transistor input decreases. Similarly a decrease in frequency degrades the signal transfer from the transistor to the load resistor.

The fact that the emitter bypass capacitor also affects the low-frequency response is perhaps less clear immediately, and it is instructive to appreciate the reason for this conclusion. The reason is that the capacitance acts not to cause but rather to remove a cause of degradation of the response. Absent the bypass capacitor the emitter resistance provides feedback one of whose consequences is a reduction in amplifier gain. The bypass capacitor is provided to allow the feedback gain reduction for long-term spurious variations, e.g., temperature variations and manufacturing tolerances, but to remove the electrical effect of the feedback on the gain at higher frequencies for preferred signals. The bypass is effective when the reactance is small compared to the emitter resistance, and less so when it is not. Thus the bypass capacitor improves the gain as frequency increases, converse to the action of the coupling capacitors.
The incremental parameter equivalent circuit of the amplifier is drawn below using a simplified hybrid-π transistor model. The intention is to estimate the frequency response, i.e., make an approximate calculation. Hence, for example, the collector resistance (Early Effect) is omitted on the argument that a circuit design with an effective transfer of collector current to the load will intrinsically reduce its importance. Similarly omit the base-width modulation resistance between base and collector. Ultimately of course a computer computation would include these second-order considerations.

Analysis of the DC amplifier conditions is left as an exercise; straightforward bias calculations as discussed before lead to a mid-band voltage gain of 56 (35 dB), $I_C \approx 1.47 \text{ ma}$ and $r_{be} \approx 2.14\text{K}\Omega$.

A precise analysis of even this single-stage circuit is not entirely unfeasible, but it is not a particularly enjoyable task. As is too often the case it leads to complex algebraic expressions with most of the factors having little quantitative consequence. There is, however, a useful way around this calculation. Instead of calculating the true zeros and poles of the circuit we can concentrate on the response itself. Thus we observe that near to the half-power frequency, i.e., close to mid-band, the circuit gain is nearly (approximately) frequency independent. If there is a dominant pole that pole by definition acts roughly independent of the effect of the other poles. And if there is not a dominant pole the influence of any one pole on the overall response (close to mid-band) is small, and again to first order the poles act independently. On that basis a commonly used approximation is to determine the circuit poles on the assumption that they are completely uncoupled, and then to estimate the 3db frequency from those poles. Keep in mind that this is not a calculation of the actual circuit poles, but rather a calculation of fictitious poles which correspond to a circuit whose frequency response near mid-band approximates that of the actual circuit.

The ‘trick’ then is to calculate the pole associated with each capacitor in turn, with the influence of all capacitors other than the one considered removed. To ‘remove’ the influence of a capacitor we simply assume, consistent with the process described, that it operates effectively in mid-band, i.e., its reactance has become small enough to have negligible effect. In effect we assume all capacitors other than the single one being considered to be short-circuit.

In general as many calculations are needed as there are capacitors, but each individual calculation involves only one capacitor. The pole corresponding to that capacitor is $\omega_p = 1/T_p$, where $T_p$ is the time constant of the circuit ‘seen’ by the capacitor. And that time constant is simply the product of the capacitance and the Thevenin resistance ‘seen’ by the capacitor.

The half-power frequency then is estimated from the formula described before

$$\omega_{3db} \approx \sum_p \omega_p \approx \sum_p \frac{1}{T_p}$$

The actual calculation of the time constants is straightforward, albeit often tedious. Thus, for example, the time constant for the input coupling capacitor is calculated from the circuit drawn below right, where the other capacitors have been replaced by short-circuits and the input source turned off. The Thevenin resistance seen looking back from the input coupling capacitor is simply the series combination of the 1KΩ source resistance, and the parallel combination of the bias resistors and the base-emitter resistor. The right side of the circuit is not involved in this calculation.

The time constant associated with the emitter bypass capacitor is calculated similarly, although a circuit transformation can make the calculation simpler. The transformation is shown below.

Verification that the terminal behavior of the two circuits is the same is left for an exercise.
The figure to the right presents the calculated time constants expressed in terms of the emitter capacitor, and the corresponding estimate of the half-power frequency.

The calculation assumes resistance in kilohns, and capacitance in microfarads; consequently time units are milliseconds, and frequency is in kiloradians/second (or KHz).

$$T_1 (\text{CIN1}) = 0.1 \cdot 1.82 \cdot 1 \cdot 0.14 = 0.2726$$
$$T_2 (\text{CIN2}) = 0.1 \cdot 5.6 \cdot 2.2 = 0.78$$
$$T_3 (\text{CE}/121) \cdot 0.56 \cdot 121 \cdot 2.14 \cdot 8.2 \cdot 10 \cdot 5.6 = 0.0426 \cdot \text{CE}$$

$$\omega_{3db} = \frac{1}{0.2726} + \frac{1}{0.78} + \frac{1}{0.024 \cdot \text{CE}} = 4.95 + \frac{41.67}{\text{CE}}$$

A PSpice analysis of the low-frequency response of the circuit (see netlist below) provided the response plotted below; computations are made for $\text{CE} = 0.5 \mu\text{F}, 5 \mu\text{F},$ and $50 \mu\text{F}$. The estimated 3db frequency for the $50 \mu\text{F}$ capacitor is $\omega_{3b} = 5.78 \text{ KRadian/second (921 Hz)}$. The computed value is about 760 Hz.

*CE Amplifier Transfer Analysis

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>VS</td>
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<td>0 AC 1</td>
</tr>
<tr>
<td>RS</td>
<td>1</td>
<td>2 1K</td>
</tr>
<tr>
<td>CIN1</td>
<td>2</td>
<td>3 .1U</td>
</tr>
<tr>
<td>R11</td>
<td>10</td>
<td>3 82K</td>
</tr>
<tr>
<td>R21</td>
<td>3</td>
<td>0 10K</td>
</tr>
<tr>
<td>Q1</td>
<td>4</td>
<td>3 5 Q2N3904</td>
</tr>
<tr>
<td>RE1</td>
<td>5</td>
<td>0 560</td>
</tr>
<tr>
<td>CE1</td>
<td>5</td>
<td>0 {CB}</td>
</tr>
<tr>
<td>RC</td>
<td>10</td>
<td>4 5.6K</td>
</tr>
<tr>
<td>VCC</td>
<td>10</td>
<td>0 15V</td>
</tr>
<tr>
<td>CIN3</td>
<td>4</td>
<td>6 .1U</td>
</tr>
<tr>
<td>RL</td>
<td>6</td>
<td>0 2.2K</td>
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<tr>
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</tr>
<tr>
<td>.PARAM CB = 50U</td>
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</tr>
<tr>
<td>.STEP PARAM CB LIST 0.5U 5U 50U</td>
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</tr>
<tr>
<td>.AC DEC 20 100 100K</td>
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<td>.OP</td>
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<td>.PROBE</td>
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<td>.END</td>
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The value of the estimation process is not primarily in the numerical calculation; a factor of two (say) between an estimate and the corresponding computed value is not particularly important for the intended purpose. Being in the ‘ballpark’ is important for some assurance of relevance, but the particular importance of the estimate procedure is that it provides information about the relative importance of different circuit elements in determining the half-power frequency. More on this point later.
**Two-Stage Illustration**

It is only a matter of more detail rather than an increase in complexity to apply the procedure to a multiple-stage amplifier. Thus the circuit diagram below shows a two-stage amplifier formed by cascading two stages of the amplifier considered above. The DC conditions are unchanged by the cascade, i.e., \( I_C \approx 1.47 \text{mA} \) and \( r_{be} \approx 2.14 \Omega \).

![Two-Stage Amplifier Circuit Diagram](image)

The incremental equivalent circuit is drawn below.

![Incremental Equivalent Circuit](image)

The several time constants are calculated in terms of unspecified values for the emitter bypass capacitors \( C_{E1} \) and \( C_{E2} \). PSpice computations for \( C_{E1} = C_{E2} = 0.5 \mu \text{F}, 5 \mu \text{F}, \) and \( 50 \mu \text{F} \) are shown below. Comparisons between the computed values and those predicted by the estimate formula are left for an exercise.

Instead we propose to make (more or less arbitrarily) \( C_{E1} = 2 \times C_{E2} \); this makes the contributions to the estimate of the half-power point of \( C_{E1} \) and \( C_{E2} \) roughly equal. Now specify the half-power point to be, say, \( 10 \text{Krad/sec} \) (1.59 kHz). A straightforward substitution indicates that we should choose \( C_{E1} = 24 \mu \text{F} \) and \( C_{E2} = 12 \mu \text{F} \). A PSpice computation (details left as an exercise) computes a 3db frequency of \( 1.12 \text{kHz} \).

If, on the other hand, we specify \( C_{E2} = 2 \times C_{E1} \), the formula specifies \( C_{E1} = 14.5 \mu \text{F} \) and \( C_{E2} = 29 \mu \text{F} \) (carry the nonstandard values for this exercise). Again the PSpice computation is left as an exercise; the computation computes a 3db frequency of \( 1.14 \text{kHz} \).

**High-Frequency Illustration**

With some additions the single-stage amplifier circuit illustration introducing the low-frequency analysis is redrawn to the right. Since the emphasis here is on the high frequency response those capacitors which
The capacitors labeled ‘\( \infty \)’ have negligible effect on the transfer response in mid-band and above, and effectively become short-circuit.

As before we assume for the time being that the transistor itself can respond to circuit voltages essentially instantaneously, and any high-frequency degradation is caused only by circuit elements. Thus the circuit includes just three capacitors representative of causes of degradation: \( C_{\text{in}} \) represents, for example, a parasitic capacitance inevitably across the input terminals. Indeed a capacitance may be added specifically because of parasitics to better define the total capacitance value, i.e., the devil you know sometimes is preferable to the devil you don’t know. \( C_{\text{in}} \) degrades the high-frequency response by making the signal transfer from the source into the transistor less effective.

\( C_{\text{i}} \) is a similar capacitance, reflecting for example the input impedance of a successor amplification stage. Both \( C_{\text{in}} \) and \( C_{\text{i}} \) may be added intentionally for some purposes, e.g. to set specific transfer function poles.

\( C_{\text{f}} \) is a particularly critical capacitor in affecting the high-frequency response because of the voltage gain of the amplifier. The amplified voltage appears across this capacitor so that the feedback current that flows is much larger in general than that corresponding to just the input voltage alone. For high-gain amplifiers this can be quite significant.

The philosophy underlying the high-frequency calculation is basically the same as for the low-frequency case. We suppose that near the (upper) 3db frequency there is weak coupling between the circuit poles, and so the response near the upper 3db frequency can be estimated by uncoupled calculation separately of each pole. Note however that a capacitance having a high-frequency effect is effectively an open-circuit at mid-band. Accordingly we calculate the time constant of each capacitor assuming that all other capacitors are open-circuit. The time constants for the illustrative circuit are drawn below (capacitance in nF, resistance in KΩ, time in microseconds, and frequency in megaHz); the 3db frequency is estimated as described before.

\[
T_1 = C_{\text{in}}[1][82][10][2.14] = 0.663\, \text{C}_{\text{in}}
\]
\[
T_2 = C_{\text{i}}[5.6][2.2] = 1.58\, C_{\text{i}}
\]
\[
T_3 = \{1|[82][10][2.14] + 1+(120/2.14)(1|[82][10][2.14])(5.6|[2.2])\} = 58.29\, C_{\text{in}}
\]
\[
\omega_{3db} = \frac{1}{0.663\, C_{\text{in}} + 1.58\, C_{\text{i}} + 58.29\, C_{\text{f}}}
\]
A PSpice computation provides the data plotted below, in which the effect of the collector-base capacitor is explored. Since the PSpice model includes an accounting for the intrinsic transistor response the curve for \( C_f = 0 \) is added to demonstrate that it is the circuit capacitances and not the transistor that are responsible for the roll-off. Also for reference a line is drawn 3db below the mid-band gain of 34.6 dB. The 3 dB frequency estimated from the formula is 546 kHz for 5pF and 54.6 kHz for 50pF.

The next plot explores the effect of \( C_{in} \) and \( C_l \).

Here also the high-frequency response for the transistor alone is provided for reference. Also for reference a line is drawn 3db below the mid-band gain of 34.6 dB. A portion of the low-frequency roll-off is included; the PSpice computation actually is made for the circuit including the ‘low-frequency’ capacitators. The 3db frequencies estimated by the formula are 71 kHz for 1 nF and 14.2 kHz for 5 nF.

**Current-Gain Bandwidth Product**

The ‘hybrid-\( \pi \)’ incremental transistor circuit model (see figure) is useful for signal frequencies up to a several tens of megahertz (depending partly on the specific device considered), after which a more detailed model becomes necessary. The base resistor \( r_{bb}' \) is added partly to account for the comparatively long internal connection from the base external connection and the actual internal base connection. A representative resistance value for this lumped resistor is in the range of 50Ω to perhaps 200Ω. This resistor ordinarily can be neglected for hand estimates. Note that \( r_{bb}' \) becomes the
dominant input resistance for frequencies so high that $C_{be}$ effectively short-circuits $r_{be}$. Ordinarily however the entire model itself generally becomes of questionable accuracy in such a circumstance.

The base-emitter junction resistance $r_{be}$ corresponds to the slope of the base diode characteristic, and $r_{ce}$ corresponds to the Early Effect. A second base-width modulation effect, characterized by a resistor connected between the base and collector is omitted; its influence is dominated by the collector junction reverse-bias capacitance $C_{bc}$.

The emitter junction (diffusion) capacitance $C_{be}$ represents the charge store to support the current flow across the base.

A parameter of some interest is the short circuit incremental current gain. This is the ratio of $i_{out}$ to $i_{in}$ when the (incremental) voltage between collector and emitter is zero. Part of the importance of this ratio is that it is in principle a function of transistor parameters only, i.e., the source and load impedance are not involved.

Straightforward analysis provides the relation

$$\text{short-circuit current gain} = \frac{i_{out}}{i_{in}} \bigg|_{v_{ce}=0} = \frac{\beta}{1 + \frac{\omega}{\omega_B}} \text{ where } \omega_B \triangleq \frac{1}{r_{be}(C_{be}+C_{bc})}$$

Experimental measurement of the corner frequency is at best awkward, and as an improvement advantage is taken of the fact that the asymptote falls off at 6 dB/octave. The product of the gain and the corresponding frequency, the gain-bandwidth product, is constant along the asymptote. The intercept at unity gain (0 dB) is then $\beta \omega_B = \omega_T$. The gain-bandwidth product does vary both with collector voltage and collector current, corresponding to changes in $C_{bc}$ and $C_{be}$ respectively.

Nevertheless $\omega_T$ serves as a measure of the frequency capability of the transistor. Note however that the circuit in which a transistor is embedded also conditions the frequency response.
PROBLEMS (low-frequency)

1) The incremental equivalent circuit for an amplifier is shown. Select a value for $C_E$ to obtain a 3db low-frequency roll-off at 1 kHz (nominal). Compare the specified and the computed response.

2) The NPN amplifier stage involves only partial bypassing of the emitter feedback resistor. Specify CE for a 3 dB frequency of 0.5 kHz. Use PSpice to compute the rolloff and compare the computed frequency to the estimated value.

3) Select values for $C_E$ and $C_i$ to obtain a 3db low-frequency roll-off a 1 kHz (nominal). Compare with the computed response. Suppose the value specified for $C_i$ is halved; specify a revised value for $C_E$ to maintain the (nominal) roll-off.

4) Select values for $C_E$ and $C_i$ to obtain a 3db low-frequency roll-off a 1 kHz (nominal). Compare with the computed response.
5) Estimate the low-frequency voltage gain (mid-band gain, 3db frequency) for the circuit shown. Then compute the response, and compare to the estimate. Q1 is 2N3904, and Q2 is 2N3906.

6) Estimate the low-frequency voltage gain (mid-band gain, 3db frequency) for the circuit shown. Compute the response, and compare to the estimate.
6) Estimate the transresistance ($V_{out}/I_{in}$) frequency response for the incremental circuit shown. Compute the frequency response and compare with the estimate.

7) Estimate the transresistance ($V_{out}/I_{in}$) frequency response for the incremental circuit shown. Compute the frequency response and compare with the estimate.

8) The 2N3904 BJT emitter current is approximately 2mA. Use this circuit in a PSpice analysis to determine a nominal value for the gain-bandwidth product.

9) Estimate the voltage gain ($V_{out}/V_{in}$) frequency response for the incremental circuit shown. Compute the frequency response and compare with the estimate. Assume $C_{bc} = 4pF$ (max value identified from manufacturer's specifications) and a (nominal) gain-bandwidth product of 350 MHz.