Objective
The time and frequency response of a linear network are characterized by a knowledge of the network singularities. Not always but surprisingly often the critical aspects of a network response are dominated by just a few of the singularities. For this reason it is useful to have some familiarity with the nature of the response of circuits with just one or two poles. Providing this basic familiarity is the objective of this experiment.

Parts List:
- opamps 2 of 741
- potentiometer 50 kΩ
- resistors 100 Ω, 1 kΩ, 2 kΩ, 2 of 3.3 kΩ, 10 kΩ, 18 kΩ
- capacitors 100 pf, 2 of 5 nf

Single Pole Response
Many opamps are designed ('compensated' is the technical description) so as to have a single dominant pole, i.e., the frequency response is dominated by the influence of just this one singularity to the exclusion of others. The pole for a compensated opamp is usually at a low frequency (≈10 Hz for the 741). The figure below illustrates a dominant-pole amplifier, together with an asymptotic representation of the gain magnitude.

An alternative description of the amplifier performance, in the time domain rather than the frequency domain, is the amplifier response to a unit step (or impulse) input. For a single-pole circuit the time response to a step input is an exponential function. 'A rise time' often used to characterize the speed of the response in reaching a steady-state is defined as the time to rise from 10% of final value to 90% of final value. It is obtained from the single-pole step-function response expression.
Note that a single-pole response involves no overshoot, i.e., there is a smooth asymptotic approach to the final value.

In the circuit drawn below an opamp is used in a noninverting voltage amplifier configuration. Generally the circuit design would insure the input impedance of the amplifier to be substantially higher than other circuit impedances, so that the input current to the amplifier can be neglected to a good approximation; for simplicity assume this is the case here. The voltage gain of the circuit, allowing for a finite amplifier gain as described before is

\[ V_o = \frac{G_o \omega_o}{s + \omega_o} \quad \frac{1}{2} = G_o \left[ \frac{1}{2} - \frac{1}{s + \omega_o} \right] \rightarrow G_o \left[ 1 - e^{-\omega_o t} \right] \]

rise time = \( \frac{\ln 2 \omega_o}{\omega_o} \approx \frac{2.2}{\omega_o} \)

The amplifier ordinarily would be designed specifically so that \( fG_0 >> 1 \) to make the midband gain essentially \( 1/f = 1 + R_2/R_1 \), i.e., depend primarily on the passive resistor values. The resistor ratio ordinarily will be designed to be greater than 1 (but \( << G_0 \)) so that the amplifier has some voltage gain. Note also that the gain expression 'with feedback' has the same form as that 'without feedback'. That is, it is a single-pole response, but with the pole displaced to a higher frequency. The gain-bandwidth product is not changed by the feedback, leading to the interesting relationship illustrated in the figure to the right; the product of the gain and frequency at any point along the 6 db per octave roll-off is the same. As increasing feedback is applied the pole frequency is increased and the gain is decreased by the same factor. A tradeoff between gain and bandwidth of some sort can be shown to be generally true for linear circuits.

**Experiment 45.1A Single Pole Response measurements**

The 741 opamp has a low-frequency gain of \( \approx 100 \) db, and a dominant pole at \( \approx 10 \) Hz. The gain-bandwidth product therefore is approximately 1 MHz. These values are obtained from the manufacturers' specifications for the 741. The test circuit shown to the right, also taken from 741 manufacturer's specifications, is used to measure the unity gain transient response data, which is then to be compared with that given as part of the device specifications. RL and CL simulate a representative amplifier load. Because of the unity-gain (voltage-follower) connection the dominant amplifier pole is moved by the feedback to a frequency equal to the gain-bandwidth product, i.e., \( \approx 1 \) MHz.

\[ V_o = \frac{G_o \omega_o}{s + \omega_o} \quad \frac{1}{2} = G_o \left[ \frac{1}{2} - \frac{1}{s + \omega_o} \right] \rightarrow G_o \left[ 1 - e^{-\omega_o t} \right] \]

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The gain-bandwidth product is \( \omega_o G_0 \)

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**Fig. 45.2** Gain-bandwidth product

**Fig. 45.3** Single-pole experimental circuit

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**Experiment 45.1A Single Pole Response measurements**

The 741 opamp has a low-frequency gain of \( \approx 100 \) db, and a dominant pole at \( \approx 10 \) Hz. The gain-bandwidth product therefore is approximately 1 MHz. These values are obtained from the manufacturers' specifications for the 741. The test circuit shown to the right, also taken from 741 manufacturer's specifications, is used to measure the unity gain transient response data, which is then to be compared with that given as part of the device specifications. RL and CL simulate a representative amplifier load. Because of the unity-gain (voltage-follower) connection the dominant amplifier pole is moved by the feedback to a frequency equal to the gain-bandwidth product, i.e., \( \approx 1 \) MHz.
Assemble the test circuit. Apply a square wave input from the function generator, with a pulse width wide enough for steady-state conditions to be established between the rising and falling pulse edges; this can be done experimentally by observing the amplifier response with the oscilloscope. Use a pulse amplitude of about 20 millivolts; this is the nominal value used for the manufacturer's tests. Measure the transient rise time ($\approx 0.5$ microsecond is the specification) and compare your measurement with the manufacturer's specifications. (Manufacturers' data indicates a small overshoot occurs, corresponding to the presence of higher-order poles of the circuit not included in the analysis.)

**Experiment 45.1B: Feedback Effects on Amplifier Response**

Modify the preceding circuit to convert the voltage follower to the noninverting amplifier drawn to the right. Derive an expression for the gain with feedback similar to that obtained for the inverting amplifier. Basically the same experiment as the preceding one is to be run, using the modified circuit. The amplifier now will have greater than unity gain, and the dominant pole will be located below 1 MHz.

Use $R_1=1\text{k}\Omega$ and $R_2=10\text{k}\Omega$ (these are nominal values; for your calculations use measured resistor values) for a nominal gain with feedback of $\approx 11$. Observe the transient response and measure the rise time.

For use in connection with transient response calculations of rise time determine the pole location from a Bode (amplitude vs frequency) plot. You will find this easier to do (and obtain more reliable results) by making a rough plot of gain (db) vs frequency (log scale); use a selected few points at low frequencies and a selected few points at high frequencies sufficient to determine asymptotes, and then extrapolate these to determine the intersection of the asymptotes. Compare the pole location obtained from frequency measurements with that obtained from transient measurements, and also to the theoretical expectation.

Repeat the experiment, changing $R_1$ to $100\\Omega$ but leaving $R_2$ at $10\text{k}\Omega$, i.e., increasing the gain by an order of magnitude.

**Two-Pole Response**

Adding a second pole to an amplifier response introduces important phenomena not present in a single-pole circuit. To investigate the two-pole response use the adjustable filter circuit drawn to the right. Assuming an idealized opamp (infinite gain, infinite input impedance, etc), a very good approximation in this experiment, the transfer characteristic of the filter is a two-pole response.

$$\frac{V_o}{V_i} = k\left[ \left( \frac{s}{\omega_0} \right)^2 + (3-k) \frac{s}{\omega_0} + 1 \right]^{-1} \quad \text{where } \omega_0 RC \gg 1$$

where $s$ is the complex frequency, and $k$ is the voltage gain of the noninverting amplifier (determined by the resistive divider in the output). Derive this equation for your report.

Note that for the resistance values specified the range of values of the gain $k$ is $1 \leq k \leq 3.78$. The pole locus in the left-hand complex plane as $k$ is varied is circular. At the lower end of this range, $k=1$, the two poles of the response coincide on the negative real axis at $\omega_0$. At $k=3$ the poles are complex conjugates on the imaginary axis at $\pm j\omega_0$. For intermediate values of $k$ the two poles are complex conjugates, with a magnitude of $\omega_0$ and a real part $= -(3-k)\omega_0/2$.

**Experiment 45.1A: Oscillation Condition**
Use nominal values of R = 3.3 k\( \Omega \) and C = 5 nf to make \( \omega_0 \) about 10 kHz. Set \( V_i \) to 0 (short-circuit the amplifier input after disconnecting the signal generator) and adjust the amplifier gain to the point where oscillations just begin; measure the frequency of oscillation \( \omega_0 \). At this point the poles have just moved onto the imaginary axis, so \( k \) should be 3. Measure the value for \( k \) at the onset of oscillation (you can use the oscilloscope to measure the resistance divider voltage ratio, and so the value of \( k \), directly).

Compare the measured values of \( \omega_0 \) and \( k \) to the theoretically expected values.

**Experiment 45.1B** Critical Damping: \( k=1 \)
For \( k=1 \) the two poles of the response are real and equal; this is the 'critically damped' case for which the step function response is

\[
\frac{V_o}{V_i} = 1 - (1+\omega_0 t) e^{-\omega_0 t}
\]

The 10% to 90% rise time for this case (obtained by plotting the transcendental expression) is \( \approx 3/\omega_0 \). Set the potentiometer for unity gain (use the oscilloscope to determine the gain) and measure the rise time. Estimate \( \omega_0 \) from this measurement and compare to the earlier measured value and to the theoretical value. Measure the frequency response (Bode plot) of the circuit and determine the location of the double pole from the asymptotic response curve; verify that the roll-off at high frequencies is 40 db per decade (or 12 db per octave).

**Experiment 45.1C**: Underdamping: \( k>1 \)
For the underdamped (\( 3 > k > 1 \)) case the poles are complex conjugates, and the two-pole response exhibits an oscillatory behavior as illustrated on the next page. The definition of several terms used in describing the response is reviewed as part of the illustration, in particular rise time (same definition as before), settling time (time period before the amplitude falls and thereafter stays within a specified error band), and overshoot (amplitude excursion in excess of the eventual steady-state value).

Some overshoot generally is accepted because the rise time becomes smaller the greater the overshoot. A common compromise designs for \( k \leq 2 \) (corresponding to complex poles located within \( \pm120^\circ \) rays in the left-hand complex plane); this corresponds to an overshoot of less than about 10%.

For this experiment the primary objective is to observe the underdamped response as \( k \) is increased. (Remember that \( k \geq 3 \) brings the circuit into oscillation.) Set \( k \approx 2 \) and measure the rise time; verify that the overshoot is modest. Compare this measured rise time to that measured for the critically damped case \( k = 1 \).
**Experiment 45.1D:** Maximally Flat Frequency Response

The description of amplifier performance complementary to a time domain description is a frequency domain description. For this experiment the emphasis is on amplitude response; the amplitude response of the two-pole function is given by

\[ \left| \frac{V_o}{V_i} \right| = \frac{k}{\sqrt{\left(1-(\frac{\omega}{\omega_0})^2\right)^2 + \left(3-k \frac{\omega}{\omega_0}\right)^2}} \]

The location of peaks in the response may be determined by differentiating the expression inside the square root and setting the result to 0 (and verifying that the extremum is a minimum; differentiate with respect to \((\omega/\omega_0)^2\) for simplicity).

In this way it can be verified that the amplitude response will have no peaks (maximum value at \(\omega = 0\) provided \(k < 3 - \sqrt{2} (=1.586)\). The response when \(k=1.586\) is called a maximally flat response and has the form

\[ \left| \frac{V_o}{V_i} \right| = \frac{k}{\sqrt{1+(\frac{\omega}{\omega_0})^4}} \]

A special significance of this particular expression is that all its derivatives are 0 at \(\omega=0\), i.e., maximally flat.

Adjust the potentiometer for unity gain, \(k=1\), and measure the frequency response (Bode plot) using a small input signal amplitude (a few tens of millivolts should do). Plot data as you take it, comparing the plot of these data with theoretical expectations. Take enough data to fill in the 'rolloff' about \(\omega_0\) adequately.

Compare the measured data against the theoretical expectation in your report. Is the response peaked? Should it be? Where is the half-power point compared to where you would expect it to be for a double pole? Is the rolloff asymptotic to 40 db/decade line?

**Experiment 45.1E:** Peaked Frequency Response

For \(k > 1.586\) the frequency response is peaked. This experiment is similar to the last one, except that the amplifier gain \(k\) is set to 2. For \(k = 2\) the response has a peak at \(\omega_0/\sqrt{2}\) the amplitude of which is 15% greater than the amplitude as \(\omega -> 0\). Measure the frequency response for this case and compare the measured response against the theoretical expectations.