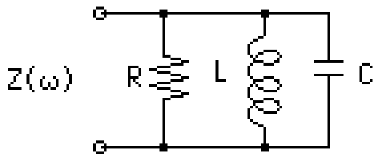
TUNED AMPLIFIER

'Tuned' amplifiers are amplifiers involving a resonant circuit, and are intended for selective amplification within a narrow band of frequencies. Radio and TV amplifiers employ tuned amplifiers to select one broadcast channel from among the many concurrently induced in an antenna or transmitted through a cable. Selected aspects of tuned amplifiers are reviewed in this note.

Parallel Resonant Circuit

An idealized parallel resonant circuit, i.e. one described by idealized circuit elements, is drawn below. The



$$Z(\omega) = \frac{1}{\frac{1}{R} + \left(j\omega C - \frac{1}{j\omega L} \right)}$$

$$= \frac{R}{1 + \frac{j}{Q^2}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}$$

where $\omega_0^2 LC \triangleq 1$ and $\frac{Q}{\omega_0 CR} = Q\frac{\omega_0 L}{R} \triangleq 1$

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* Parallel Tuned Circuit

Iin 1 0 AC 1
RT 1 0  {RVAL}
CT 1 0  1
LT 1 0  1

.AC LIN 1000 .1 .3
.PARAM RVAL = 10
.STEP PARAM RVAL LIST 50, 100, 200
.LIB EVAL.LIB
.OPTIONS NOPAGE NOMOD
.PROBE
.END
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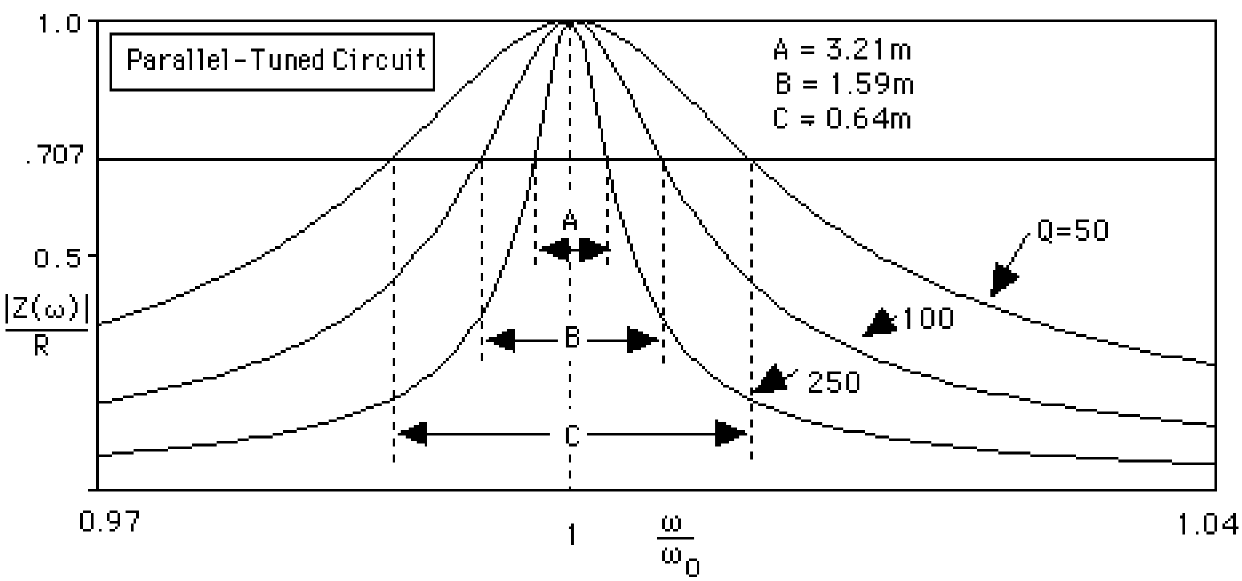
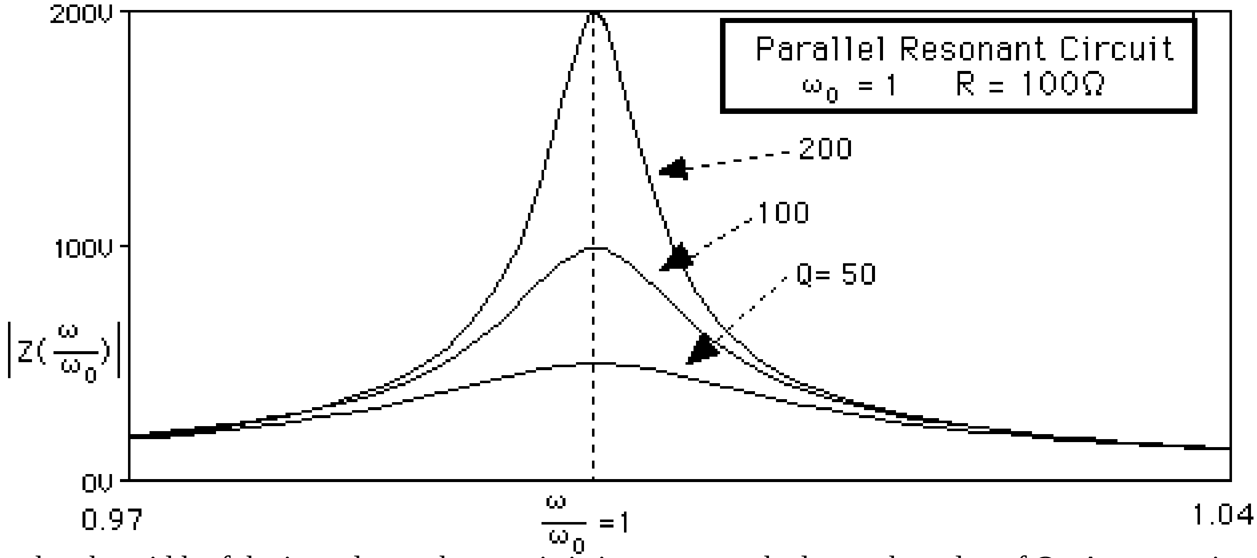
input impedance of this configuration, shown below the circuit diagram, is readily obtained. A modest algebraic restatement in convenient form also is shown. The significance of the definitions of the 'quality factor' Q and the resonant frequency ω_0 will become clear from the discussion. The influence of the Q parameter on the tuned-circuit impedance for several values of Q is plotted below for a normalized response.



As a complementary numerical illustration the netlist to the right of the circuit diagram (see above) is used to compute the input impedance as a function of (radian) frequency, for three values of Q. The maximum impedance magnitude occurs when the imaginary part of the denominator (in the expression for $Z(\omega)$ is zero, i.e., at the resonant frequency ω_0, and this impedance is equal to R. These characteristics may be observed in the plot.



Note that the width of the impedance characteristic is narrowest the larger the value of Q. As convenient measure of the narrowness is the 'bandwidth', defined as the difference in frequency between the points where the impedance magnitude is R√2/2, i.e. 3 db lower than the peak value R. These points correspond to equal real and imaginary parts in the denominator of the impedance expression, i.e.

$$\frac{1}{Q^2}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2 = 1$$

This equation may be expressed as

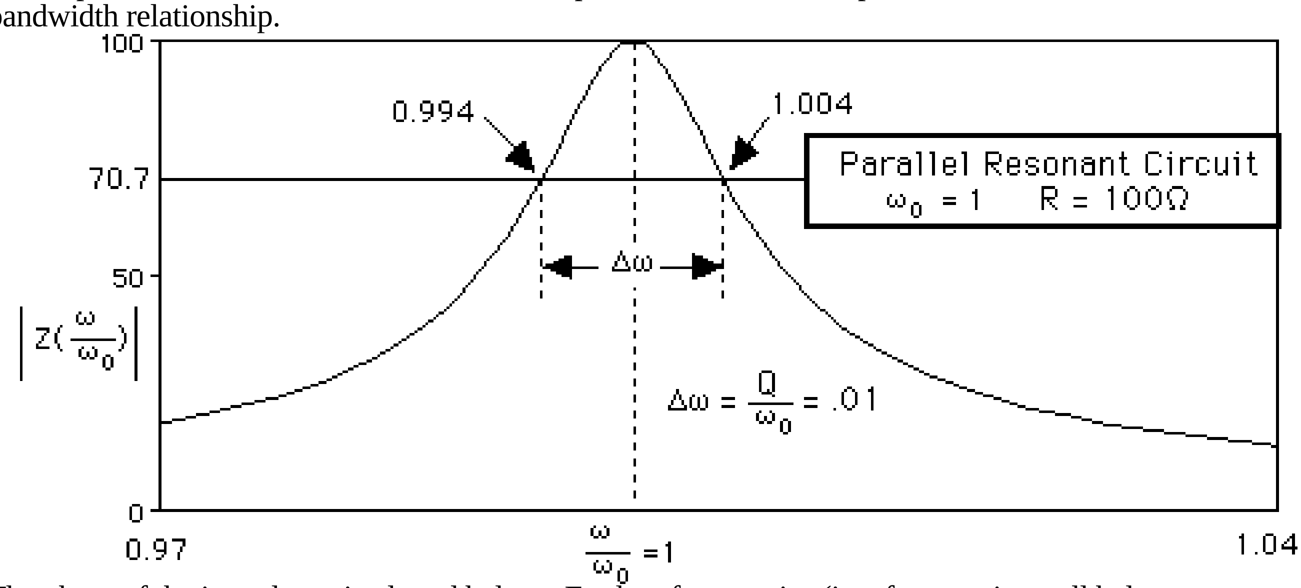
$$\left[\frac{1}{Q}\,\frac{(\omega-\omega_0)(\omega+\omega_0)}{\omega_0\omega}\right]^2 = 1$$

If the bandwidth is indeed narrow (Q > 10 or so), as would be all but certain in a useful application, then $\omega << \omega_0$ at the half-power points, and $\omega + \omega_0 \approx 2\omega_0$, and $\omega_0\omega \approx \omega_0{}^2$ Substituting these approximations obtain

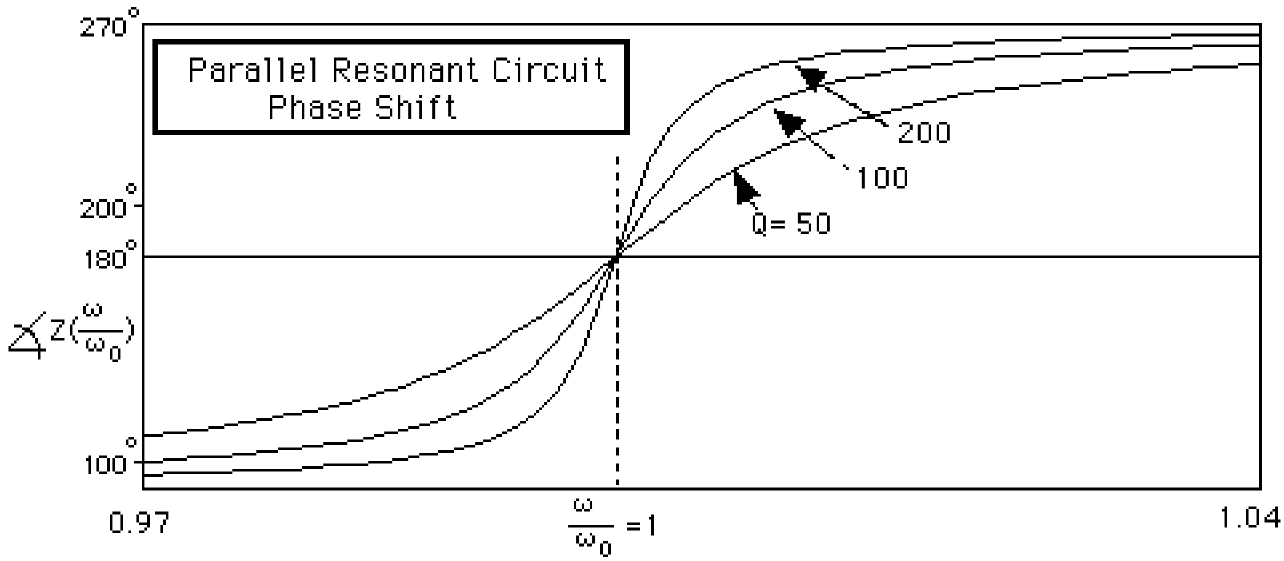
$$\omega - \omega_0 = \pm\frac{\omega_0}{2Q}$$

$$\text{3db bandwidth } \Delta\omega = \frac{\omega_0}{Q}$$

The impedance curve for the case R = 100Ω is plotted below on an expanded scale to illustrate the bandwidth relationship.

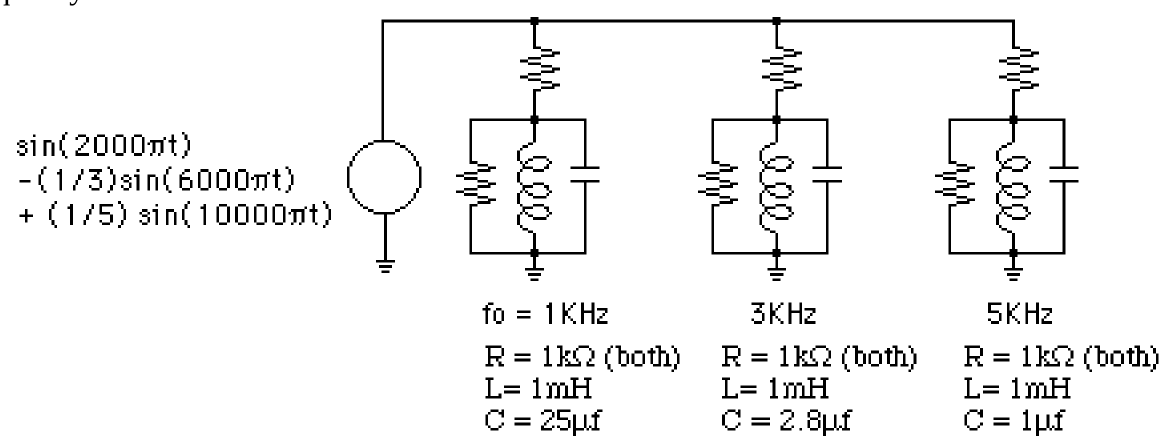


The phase of the impedance is plotted below. For low frequencies (i.e., frequencies well below resonance the inductor dominates, and the phase shift approaches 90˚. Conversely at frequencies well above resonance the capacitor dominates the impedance and the phase shift approaches -90˚ (or +270˚); resonance marks zero phase shift. Note that the phase shift changes from one extreme to the other essentially within the bandwidth.



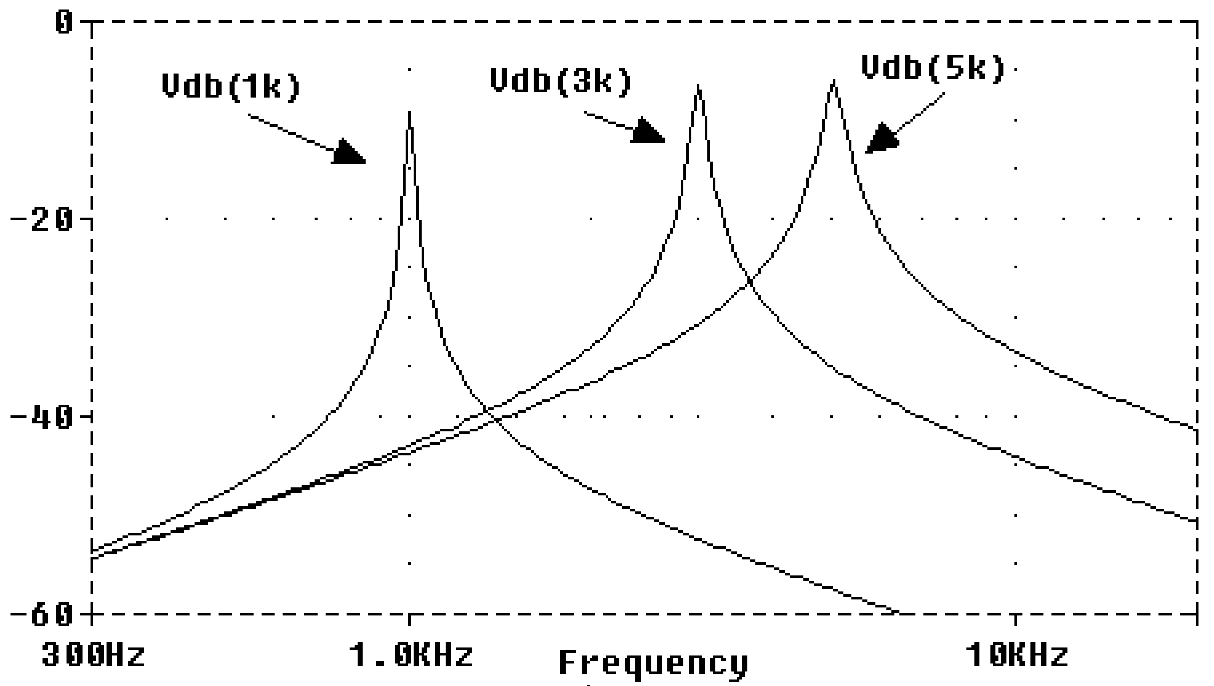
Frequency-Sensitive Voltage Divider

To illustrate the selectivity of the tuned circuit the circuit drawn below is analyzed. Each branch consists of a voltage divider formed by a resistor in series with a parallel resonant circuit. The voltage across each tuned circuit is a maximum at resonance; resonance occurs at different frequencies in each case, as noted. The voltage driving the circuit is a superposition of sine waves, actually the first three terms of a Fourier expansion of a square wave. The resonant circuits are designed to select respectively the fundamental frequency and the two harmonics.

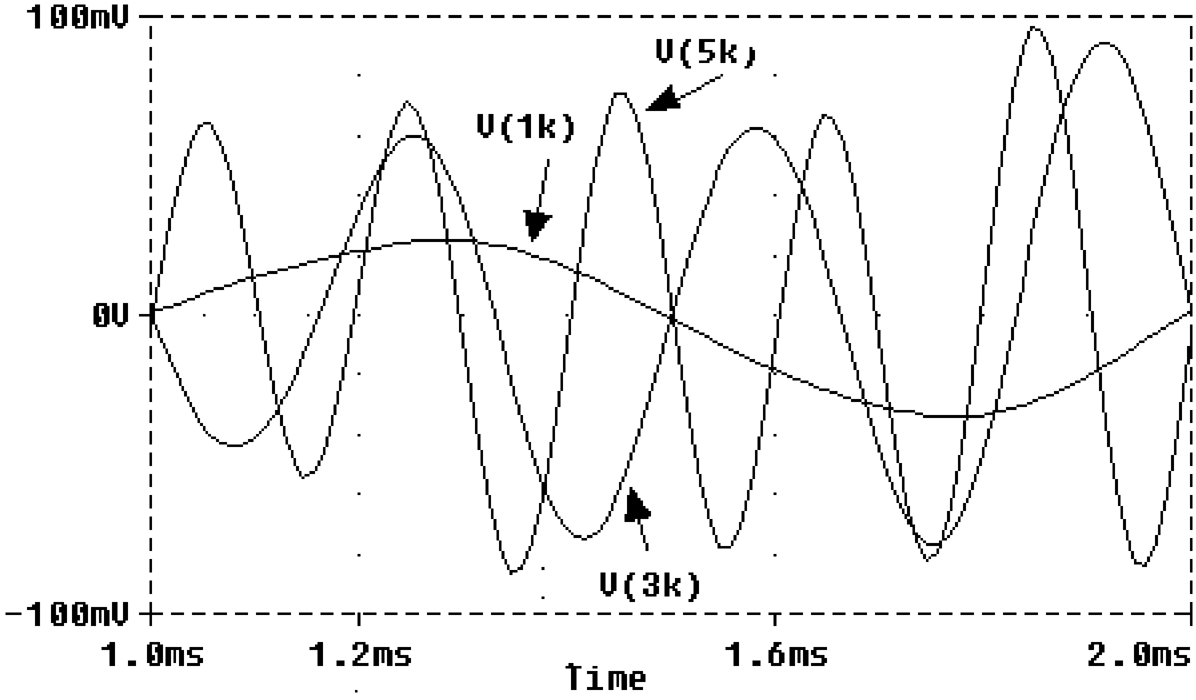


To analyze the circuit response apply (conceptually) superposition; each sinusoidal component of the input signal is considered separately, and the individual responses calculated superimposed. The computed voltage across each tank is plotted below to illustrate the resonant frequency discrimination. Note that the Q is 158.1 (1KHz circuit), 52.9 (3KHz circuit), and 31.6 (5KHz circuit).

These data are presented as Bode plot s to accentuate the rapid attenuation off resonance. Thus note that Vdb (5), the voltage across the 1 kHz resonant circuit, is down some 40 db, a factor of 100, from its resonant amplitude at 3 kHz. The voltage divider action discriminates against signals with frequencies off resonance. Similarly the 3 kHz tank 'prefers' signal frequencies in a narrow frequency range about 3 kHz.



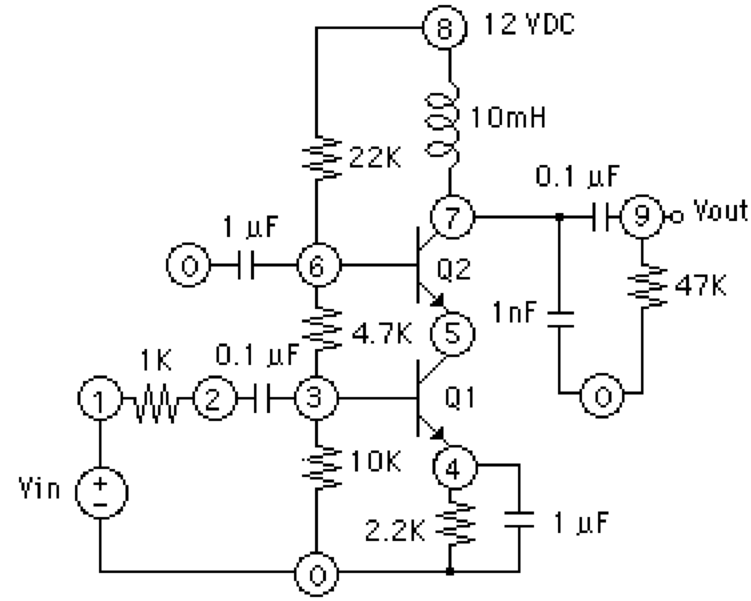
To further indicate the frequency discrimination the voltage across each tuned circuit is plotted below as a function of time. Note the comparative purity of the sinusoidal waveforms, indicating significant rejection of other than the resonant frequency.



Tuned Circuit amplifier Example

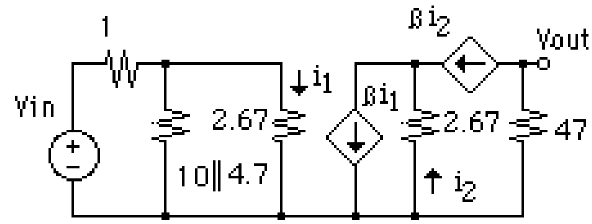
A two-stage 'cascode' tuned amplifier circuit is drawn below. The input stage is a CE configuration whose collector current is the input to a CB output stage. This configuration provides some advantages in preventing oscillation but for the present this aspect of the circuit is not considered.

The LC product is (10mH)(1nF) = 10^{-12}, and so the resonant frequency is 50.3 kHz. The Q of the resonant circuit is $\omega_o RC$ = 14.8, and so the bandwidth is ω_o/Q = 3.4 kHz.



Assume, subject to verification, that the Q1 and Q2 base currents are small compared to the current in the biasing resistors; estimate this current as 12/(22 + 4.7 + 10) = 0.33 ma. Then V(3) is approximately 3.3 volts, and V(4) about 0.7 volt less. The Q1 emitter current is then about 2.6/2.2 = 1.18 ma. With a nominal 2N3904 ß = 120 the base current is about 9.8 µa, and this is reasonably small compared to 0.33 ma. The Q2 currents are, closely enough, the same as for Q1. Determine r_{be} = (121)(26)/(1.18) = 2.67 K.

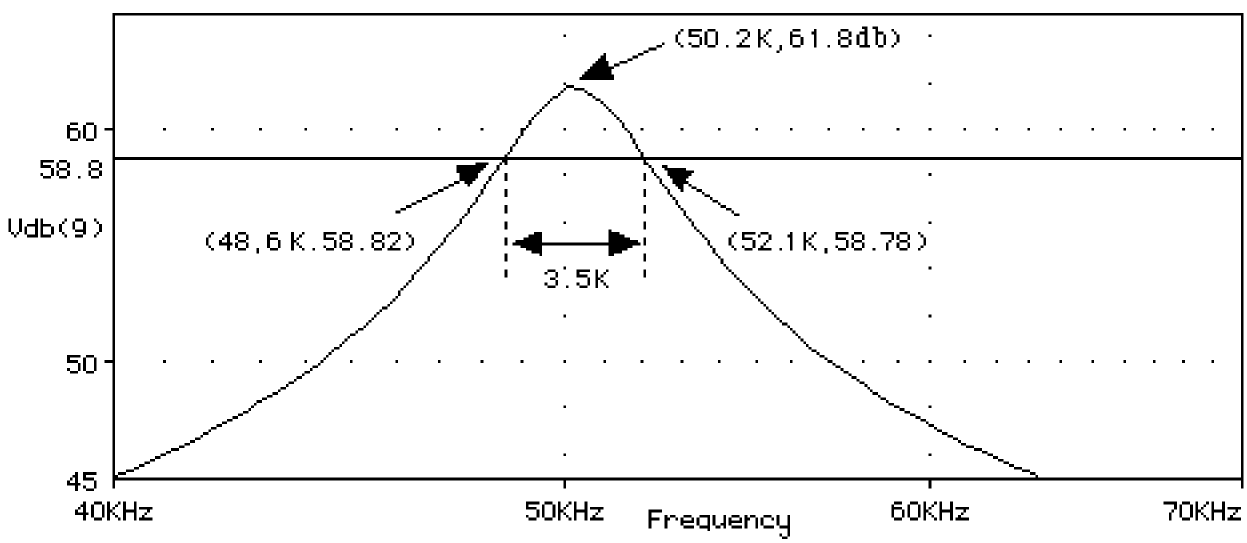
At the resonant frequency the coupling and emitter bypass capacitors are effective (verify this), and the resonant circuit is resistive. The incremental equivalent circuit in resonance is



as shown to the right, and the gain calculation is

$$\frac{Vout}{Vin} = \frac{1}{1+10\|4.7\|2.67} \; \frac{10\|4.7}{(10\|4.7)+2.67} \; (-120) \; \frac{120}{121} \; 47$$
$$= 1238 \text{ (61.8 db)}$$

A plot of the gain characteristic computed by PSpice follows. Comparison of the computed behavior with the predicted behavior is left as an exercise.

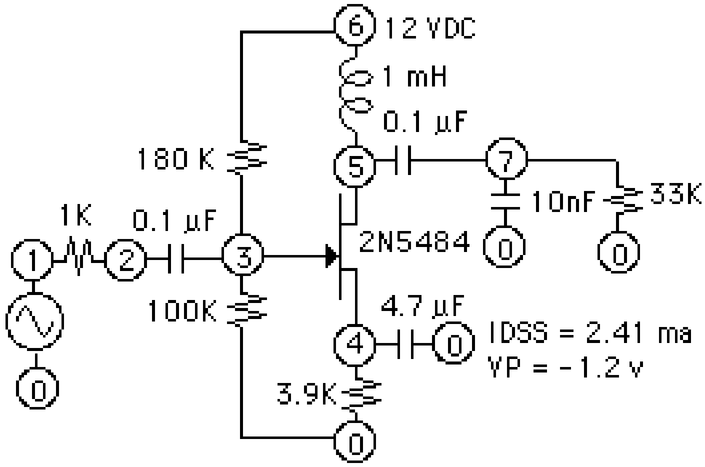


<u>FET Tuned Amplifier; Another Illustration</u>

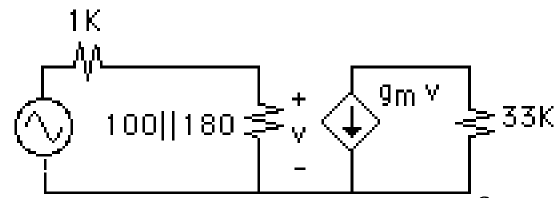
A straightforward CE tuned amplifier configuration provides a second example; the circuit diagram is drawn below, right. An estimate of the amplifier performance is made first, using a first-order JFET model for the 2N5484 (IDSS = 2.41 ma, VP = -1.2v). Thus write

$$ID = 2.41\left(1 + \frac{VGS}{1.2}\right)^2$$
and
$$ID = 4.29 - 3.9ID$$

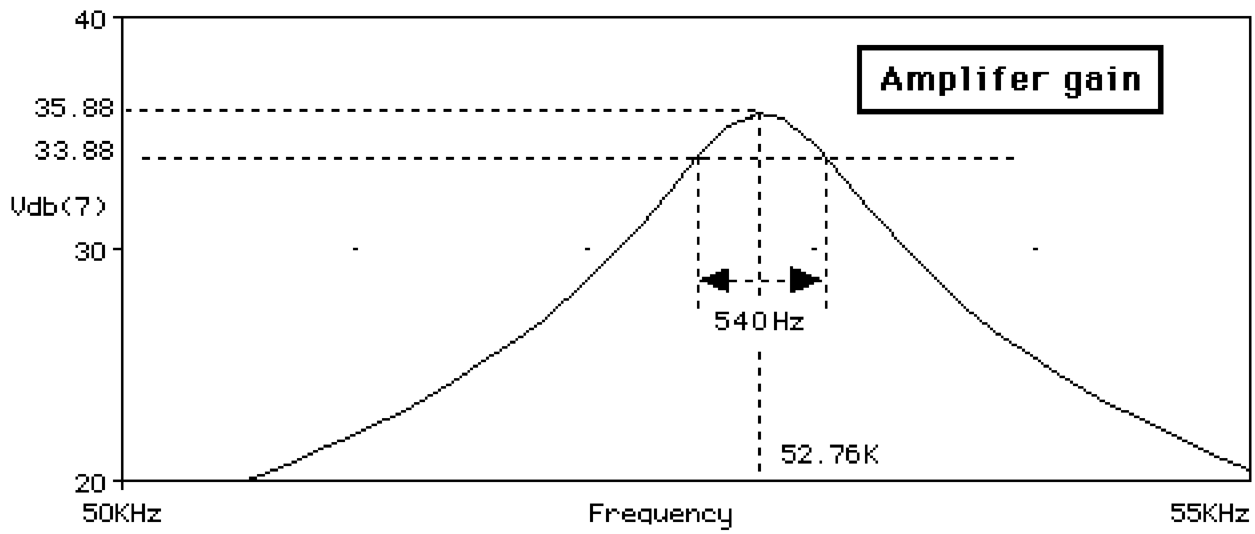
where the gate voltage is 4.29v. Solve for the quiescent bias values ID = 1.19 ma and VGS = –0.35v. A PSpice computation provides VGS = -0.4 v and ID = 1.24 ma. Note that the first-order model does not allow for Channel Length Modulation.



The incremental parameter circuit for operation at resonance is drawn below; the JFET model ignores channel-length modulation. Using the calculated bias current and voltage values the transconductance gm is 2.84 ma/v. The voltage gain at resonance then is 92.3 (39.3 db).



The calculated resonant frequency is $10^6/2\pi$ = 50.35 kHz. The calculated Q is approximately 104 and the 3db bandwidth therefore is estimated to be 484 Hz. PSpice is used to provide the numerical analysis of the amplifier performance plotted below.

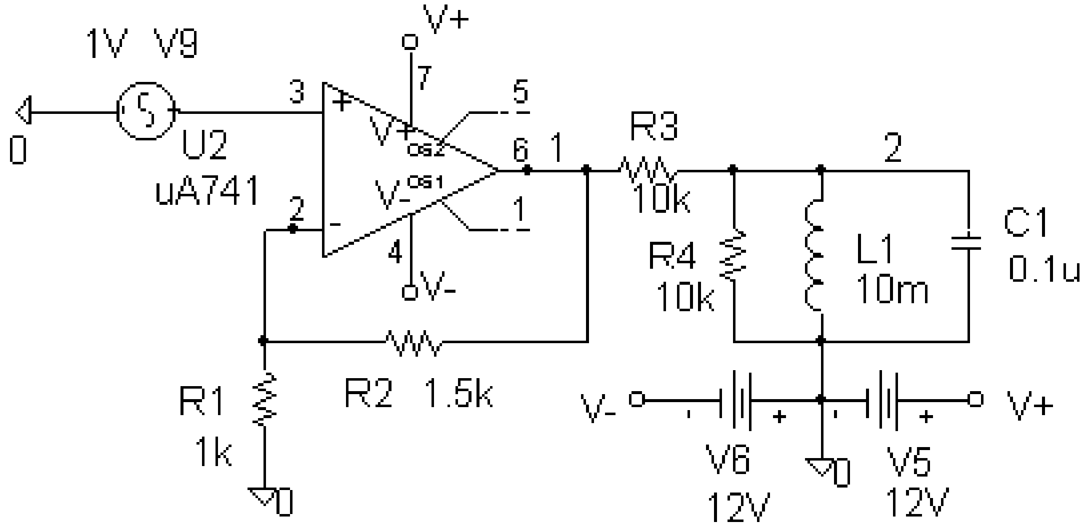


Tuned Circuit Harmonic Oscillator

The sine wave has a fundamental association with many phenomena; here it is periodic phenomena in particular that are considered. Any non-pathological periodic function may be represented as a superposition of sine waves whose frequencies are integral multiples of a fundamental frequency. This is of course the Fourier Series representation of a periodic function.

(One type of harmonic oscillator may be assembled as a regenerative feedback amplifier circuit. The amplifier replaces energy caused by inevitable circuit losses, since absent the replacement there would eventually be no energy left to sustain an oscillation. However the amplifier does not control the oscillation amplitude. If there is an increase in output amplitude, and there must be at least initially, there will be a greater feedback, so a greater output, … . The growth is limited eventually by some nonlinearity, e.g. amplifier saturation. This reduces the loop gain, the feedback signal amplitude becomes inadequate to sustain the oscillation, and oscillation stops. This brings the amplifier out of saturation, gain is restored, and oscillation restarts, driving the system into saturation again. There is hence a periodic signal generated that can be described as a superposition of sinusoids.

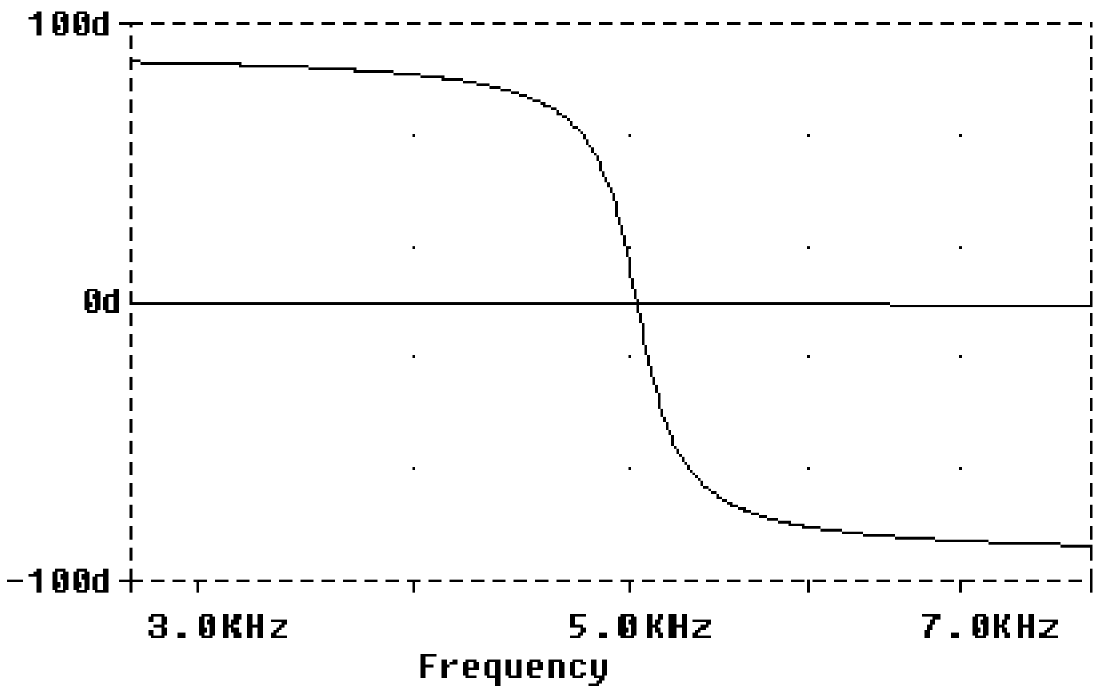
One other circuit mechanism is needed, and this is a means of fixing the fundamental oscillation frequency. Of the several commonly used methods used to do this one, a tuned circuit, is described here. A simplified oscillator circuit is drawn below.



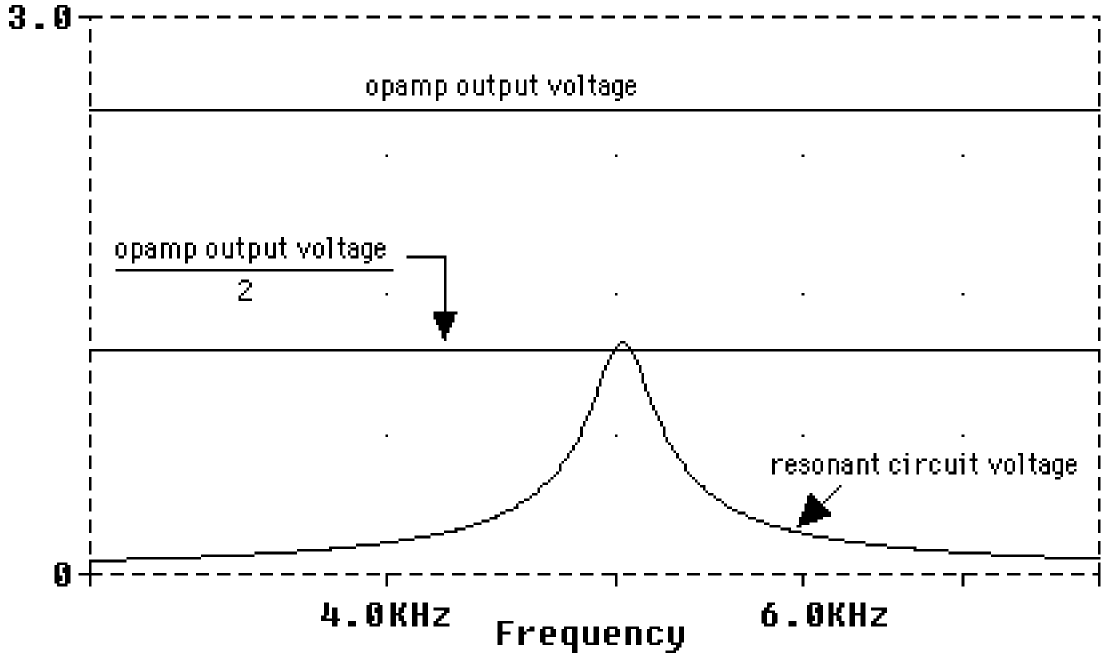
The frequency sensitive circuit to be used to establish the oscillation frequency is a parallel resonant circuit as shown on the right of the figure. The voltage tapped off the RLC 'tank' will be fed back to the input of an (idealized) opamp to provide a signal gain (idealized) of 2.5. For the moment however an AC signal source is used to examine certain circuit behavior.

The amplified signal is applied across a frequency-sensitive voltage divider formed by R3 and the parallel resonant circuit. At resonance the voltage across the tank would be 2.5/2 = 1.25 the amplitude of the original signal fed to the opamp input. Hence a larger signal of the proper phase will be fed back on the next pass, and the regenerative action will continue.

For the parameter values shown the resonance frequency (fundamental signal frequency) is 5.03 kHz. The computed plot of circuit phase shift follows. The amplifier operates in its midband frequency range where it contributes negligible phase shift. The resonant circuit phase shift is zero (resonance) at the expected frequency.



The associated open-loop amplitude plot ifollows. Note that the amplitude of the voltage across the resonant circuit at resonance is about half the opamp output voltage, as expected.



The output of the closed loop circuit (source replaced by the feedback connection) is shown below. Note the saturated operation of the opamp. Notice also the relatively clean sinusoidal oscillation across the resonant circuit. Recall that the opamp output is across a frequency-sensitive voltage divider, and that as the frequency increases the fraction of the opamp output voltage that appears across the tank is smaller. Examine the resonant circuit amplitude response drawn above and note that there is a substantial decrease even at the second harmonic of the saturated waveform.