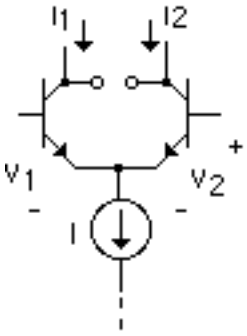
Triangle to Sine Conversion
(Nonlinear Function Fitting)

Elsewhere piecewise-linear diode approximations to nonlinear functions, and in particular triangle-to-sine conversion, have been described. Although more involved (in general) function-fitting using nonlinear functions also can be done. A case in point is a triangle-to- sine conversion illustrated in a National Semiconductor application note 'Sine Wave Generation Techniques' AN-263 (March 1981) using the logarithmic characteristic of a differential amplifier.



1) $V = \frac{kT}{q_e} \ln \frac{I}{I_o}$

2) $V_1 - V_2 = \frac{kT}{q_e} \ln \frac{I_1}{I_2}$

3) $x \triangleq (V_1 - V_2)\frac{q_e}{kT}$

4) $\frac{I_1}{I_2} = e^x$

5) $I = I_1 + I_2$

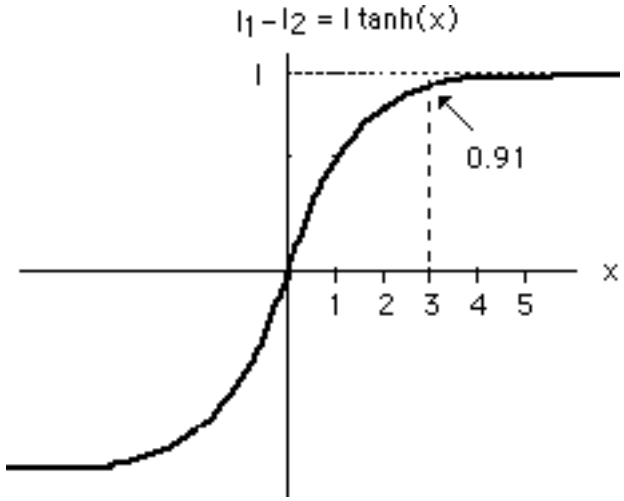
6) $\frac{I_1}{I} = \frac{1}{1+e^{-x}} \quad \frac{I_2}{I} = \frac{1}{1+e^{x}}$

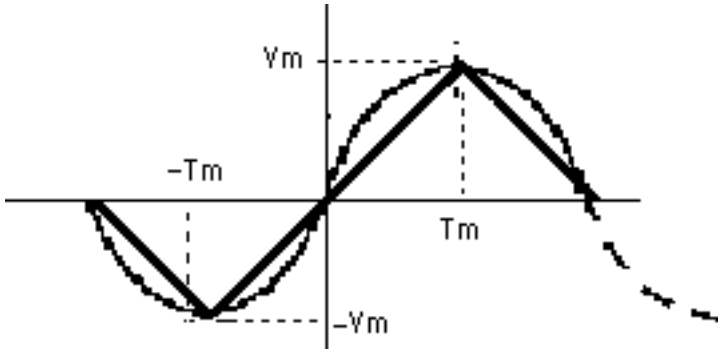
7) $\frac{I_1 - I_2}{I} = \frac{1}{1+e^{-x}} - \frac{1}{1+e^{x}}$

$= \tanh(x/2)$

The figure on the left is an abbreviated derivation of the pertinent expression. A plot of the hyperbolic tangent is drawn below. Note that this derivation does not require incremental-signal assumptions; it assumes only the well-established exponential junction behavior and (for simplicity) ß >> 1.

Note that within a limited range (say –3 < x < 3) the hyperbolic tangent at least arguably appears to be sinusoidal. Additional semi-quantitative support for this assertion follows with the following reasoning. Imagine the range from (say –3 ≤ x≤ 3) to correspond to a half-cycle of a periodic function by applying a triangular waveform. This is illustrated below; the triangular wave is applied as the base signal of the differential pair, with the peak values ±Vm selected appropriately .





If the periodic function so formed is expanded in a Fourier series that series contains only sine terms since the function is odd. Moreover the odd symmetry indicates that no even harmonics will be involved;.

The triangular wave is of course periodic with period 2Tm. As a not-entirely crude function-fitting simplification suppose we require both the hyperbolic tangent and the sinusoid to have about the same slope at t = 0 (both functions are approximated by their arguments for small t). This requires selecting Vm such that $\pi = qeVm/kT$, or Vm ≈ 81.6 millivolts at 300K.

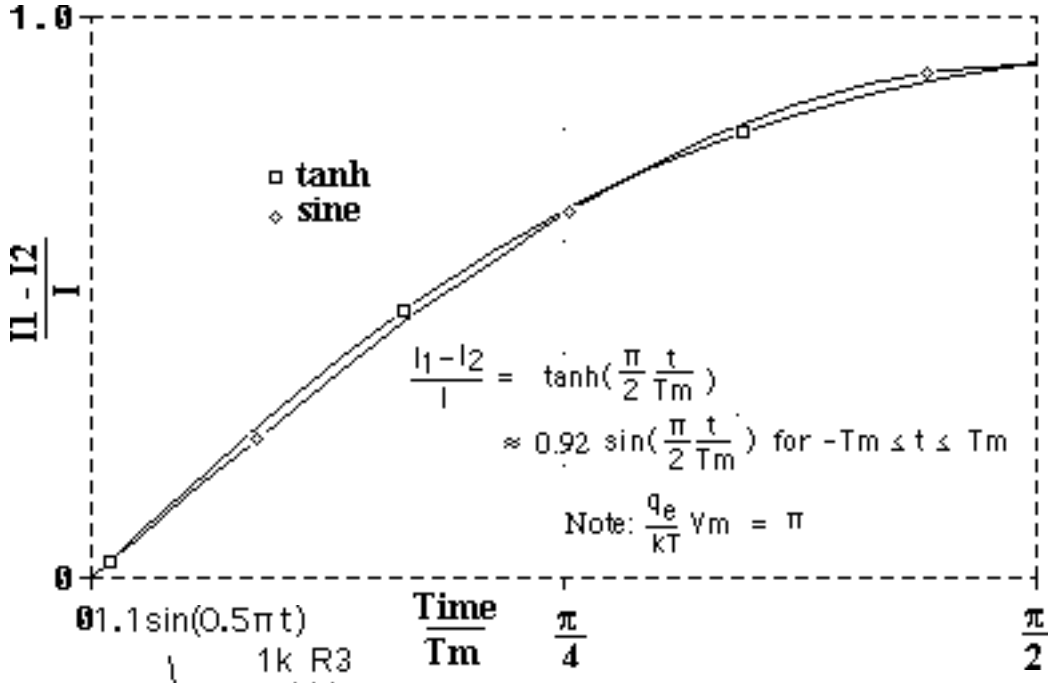
$$I_1 - I_2 = I \tanh\left(\frac{1}{2}\frac{q_e}{kT}\frac{Vm}{Tm}t\right)$$

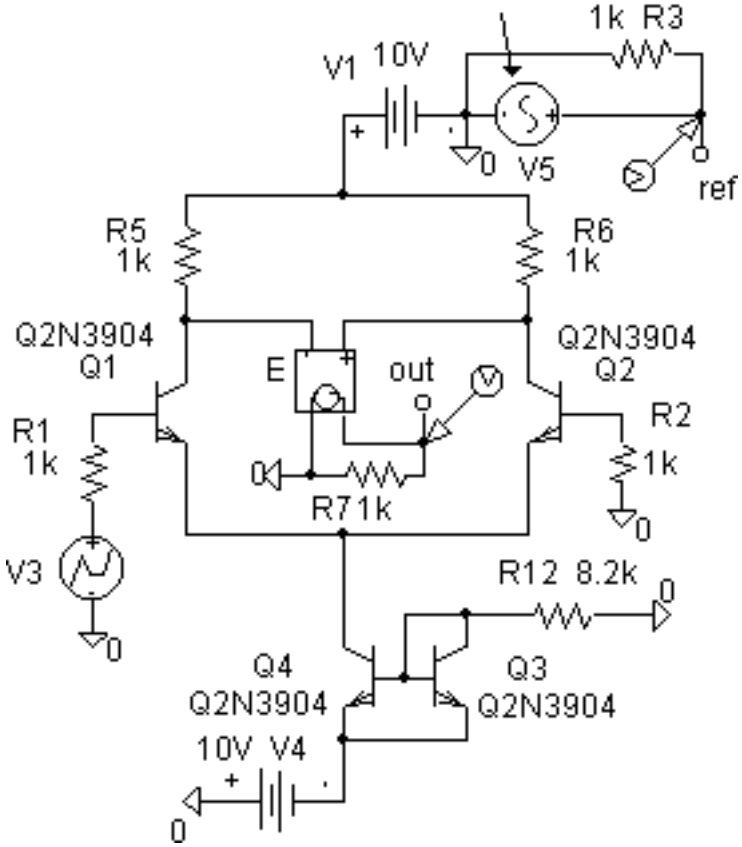
$$\approx I \sin\left(\frac{\pi}{2}\frac{t}{Tm}\right) \quad \text{for } -Tm \le t \le Tm$$

Similarly, again as an intuitive 'fitting', suppose we chose the peak value of the sinusoid and the value of the hyperbolic tangent to be the same at t = Tm (note that tanh(π/2) = 0.92). (This reduces the initial slope of the sinusoid about 5%, a sophistic adjustment ignored here.)

This assures the two functions have equal values at the origin and at their peak, and both start with very nearly the same slope; a sort of general continuity in nature suggests that the relationship between corresponding intermediate points is improved in consequence.

The two functions are compared in the figure to the left; even the unrefined approach taken is seen to yields a surprisingly good approximation. Note the 'crossover' of the two functions.

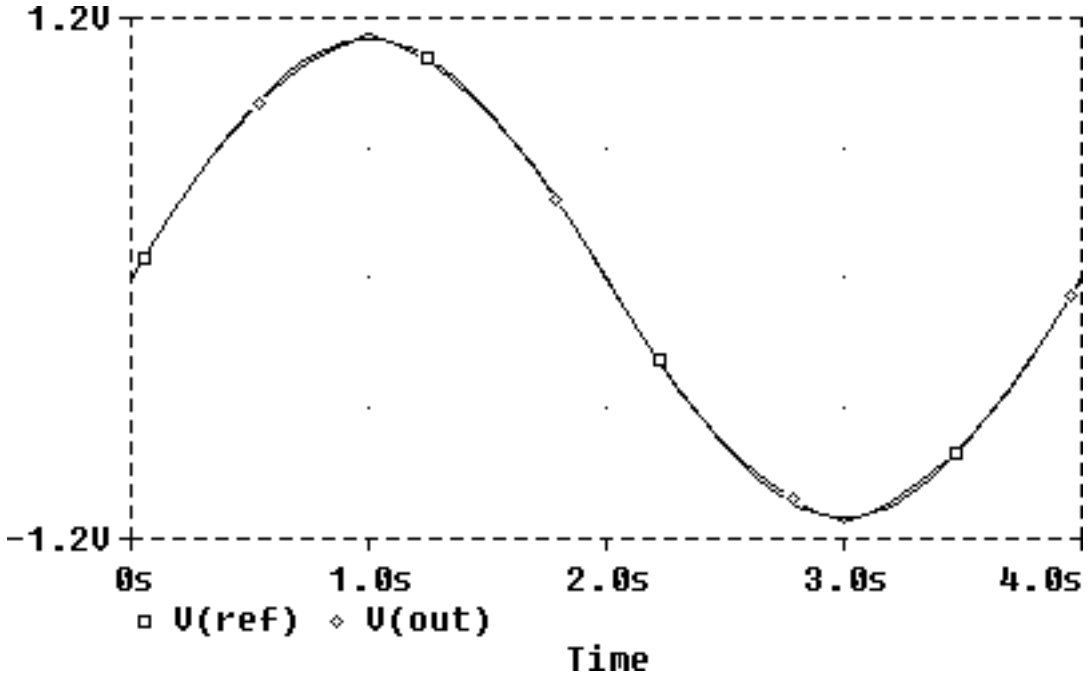




To test the fit in practice the differential amplifier test circuit drawn below was assembled. The base voltage is a triangular waveform with varying between ± 85 millivolts (corresponding to the approximate slope assumption described before), and with a (more or less arbitrarily chosen) normalized period of 4 seconds. Rather than monitoring the differential currents the differential collector voltage is used. The emitter bias current provided by Q4 is ≈ (10 – 0.7)/8,2 = 1.13 ma. For latter purposes of comparison a 'reference' sinusoidal voltage source with amplitude 1.1 (1.1ma*1kΩ) is defined at the upper right of the circuit. As a simplification a voltage-controlled voltage source E is used to extract the differential collector voltage.

A PSpice computation of the differential output voltage is plotted below, and compared to the sinusoidal reference. Total harmonic distortion of the output waveform (10

harmonics) is computed to be 1.2%.



Another comparison of some interest is plotted next; this is a comparison of the triangular waveform against the associated ‘sinusoidal’ output.