Triangle to Sine Conversion
(Nonlinear Function Fitting)

Elsewhere piecewise-linear diode approximations to nonlinear functions, and in particular triangle-to-sine conversion, have been described. Although more involved (in general) function-fitting using nonlinear functions also can be done. A case in point is a triangle-to-sine conversion illustrated in a National Semiconductor application note ‘Sine Wave Generation Techniques’ AN-263 (March 1981) using the logarithmic characteristic of a differential amplifier.

The figure on the left is an abbreviated derivation of the pertinent expression. A plot of the hyperbolic tangent is drawn below. Note that this derivation does not require incremental-signal assumptions; it assumes only the well-established exponential junction behavior and (for simplicity) β >> 1.

Note that within a limited range (say –3 < x < 3) the hyperbolic tangent at least arguably appears to be sinusoidal. Additional semi-quantitative support for this assertion follows with the following reasoning. Imagine the range from (say –3 ≤ x ≤ 3) to correspond to a half-cycle of a periodic function by applying a triangular waveform. This is illustrated below; the triangular wave is applied as the base signal of the differential pair, with the peak values ±V_m selected appropriately.

If the periodic function so formed is expanded in a Fourier series that series contains only sine terms since the function is odd. Moreover the odd symmetry indicates that no even harmonics will be involved:

The triangular wave is of course periodic with period 2T_m. As a not-entirely crude function-fitting simplification suppose we require both the hyperbolic tangent and the sinusoid to have about the same slope at t = 0 (both functions are approximated by their arguments for small t). This requires selecting V_m such that π = q_eV_m/kT, or V_m ≈ 81.6 millivolts at 300K.
Similarly, again as an intuitive ‘fitting’, suppose we chose the peak value of the sinusoid and the value of the hyperbolic tangent to be the same at \( t = T_m \) (note that \( \tanh(\pi/2) = 0.92 \)). (This reduces the initial slope of the sinusoid about 5%, a sophistic adjustment ignored here.)

This assures the two functions have equal values at the origin and at their peak, and both start with very nearly the same slope; a sort of general continuity in nature suggests that the relationship between corresponding intermediate points is improved in consequence.

The two functions are compared in the figure to the left; even the unrefined approach taken is seen to yield a surprisingly good approximation. Note the ‘crossover’ of the two functions.

To test the fit in practice the differential amplifier test circuit drawn below was assembled. The base voltage is a triangular waveform with varying between ± 85 millivolts (corresponding to the approximate slope assumption described before), and with a (more or less arbitrarily chosen) normalized period of 4 seconds. Rather than monitoring the differential currents the differential collector voltage is used. The emitter bias current provided by Q4 is \( \approx (10 - 0.7)/8.2 = 1.13 \text{ ma} \). For latter purposes of comparison a ‘reference’ sinusoidal voltage source with amplitude 1.1 (1.1mA*1kΩ) is defined at the upper right of the circuit. As a simplification a voltage-controlled voltage source \( E \) is used to extract the differential collector voltage.

A PSpice computation of the differential output voltage is plotted below, and compared to the sinusoidal reference. Total harmonic distortion of the output waveform (10...
harmonics) is computed to be 1.2%.

Another comparison of some interest is plotted next; this is a comparison of the triangular waveform against the associated ‘sinusoidal’ output.