Visible sunlight consists of a continuous spectrum of colors ranging from violet to red. The visible wavelengths range from just below 400 nm (violet) to well over 700 nm (red).
A balanced (i.e. flat) spectrum appears white to the human observer. An unbalanced spectrum exhibits some shade of color to the human observer. A human observer perceives color through the stimuli of three different pigmented cones in the human eye. These cones are shortly denoted red, green and blue (or long, medium, and short).

Two (different) spectra that produce exactly the same responses of the cones, i.e. the tristimuli, are said to have the same color and are indistinguishable for the human observer.

Achromatic (void of color) light’s only attribute is intensity.

The response to the three cones can be modeled by a three-vector \( c \), with

\[
c_i = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} s_i(\lambda)f(\lambda)\,d\lambda \quad i = 1, 2, 3
\]

After sampling the spectra (10 nm) we get in matrix notation

\[
c = S^T f
\]

with \( c \) (3x1): the cone response vector,
\( S \) (Nx3): sampled version of the spectral sensitivity functions \( s_i(\lambda) \), \( S = [s_1, s_2, s_3] \)
\( f \) (Nx1): sampled version of the spectrum \( f(\lambda) \).

Two different spectra, represented by different \( N \)-vectors, \( f \) and \( g \) produce the same response vector and therefore represent the same color if

\[
S^T f = S^T g
\]

For tri-chromacy we choose three primaries, \( P = [p_1, p_2, p_3] \)

\[
\text{colorimetric independence if: } S^T p_i, S^T p_j, S^T p_k \text{ are linear independent}
\]

A color match is achieved or any three-vector \( a(f) \) that satisfies

\[
S^T P a(f) = S^T f
\]

The matching of all \( N \) monochromatic spectra \( e_i \) yields

\[
S^T P e_i = S^T e_i \quad i = 1, \ldots, N
\]

\[
S^T P a' = S^T f
\]

with \( A \) (Nx3) the color-matching matrix for the primaries \( P \)

\[
A = (P^T S)^{-1}
\]

A weighted sum of primaries produces a color that cannot be distinguished by a standard observer from the color of a spectrum \( f \).

\[
S^T P a(f) = S^T f
\]

Weighted sum of primaries

Standard observer
CIE RGB CMF's

- CIE RGB Color Matching Functions (CMF's) are defined experimentally.
- Primaries are monochromatic sources at: 700.0 nm, 546.1 nm, 435.8 nm with radiant intensities that yield equal tri-stimulus for an equi-energy spectrum.
- CMF's: \( R(\lambda), G(\lambda), B(\lambda) \) can become negative!
- How can we get negative intensities in color matching experiments?

CIE XYZ CMF's

- CMF's: \( X(\lambda), Y(\lambda), Z(\lambda) \) are linear transformations of CIE RGB CMF's
- Constraints:
  - non-negative spectra,
  - \( Y(\lambda) \) should be coincident with the luminous efficiency function
  - normalization: equal tri-stimulus values for an equi-energy spectrum

Standard Illuminants

- The observed spectra from (non-luminous), diffuse reflecting objects depend heavily on the illuminant and characterized by the reflectivity vector \( r \) \((0 < r < 1)\)
- The CIE XYZ tristimulus values are:
  \[ t = A' L r = A' r \]
  with \( A \) the matrix of CIE XYZ CMF's, \( L \) the diagonal illuminant matrix, and \( A' \) the visual subspace of \( L \).

Chromaticity coordinates

- CIE XYZ chromaticity coordinates:
  \[ x = \frac{X}{X + Y + Z} \]
  \[ y = \frac{Y}{X + Y + Z} \]
  \[ z = 1 - x - y \]
- All visible monochromatic spectra appear on the horse-shoe.
- All mixtures appear inside
Transformation of primaries

- Standardized sets of RGB primaries appear in TV sets (NTSC / PAL / HDTV)
- Assume primaries $Q$ and corresponding CMF's $B$

$$B' = (A'Q)^{-1}A'$$

- The matrix $(A'Q)^{-1}$ is also used to transform tristimuli values in the primary system $P$ to the tristimuli in the system $Q$.

RGB color model

- RGB model uses the primaries Red, Green and Blue to produce a color.
- The spectral components of red, green, and blue are added.

HSI color model I

- HSI is suitable for describing the colors in the RGB cube
- Hue: describes a pure color (red-magenta-blue-cyan-green-yellow-red)
- Saturation: a measure for the amount of dilution by achromatic light
- Intensity: is related to brightness

- RGB-cube with the Black-White axis in upright position.

HSI color model II

- Notice that the slices are not perceptually uniform.
**CMY / CMYK color model**

- The CMY color model consists of the secondary colors of emitted light.
- CMY are the primary colors of pigments.
- The pigments absorb certain wavelengths of the illumination source and subtract these wavelengths from the reflected light.
- Simple conversions such as: 
  \[
  C = 1 - R, \quad M = 1 - G, \quad Y = 1 - B,
  \]
  will not produce a faithful reproduction of an RGB encoded color scene by a printer on paper.
- The reproduction of a specified color in XYZ by a CMY device requires careful calibration, resulting in nonlinear transforms.

**Uniform color spaces**

- Perceptually uniform color spaces are: CIE \( L^*a^*b^* \) (and CIE \( u^*v^*w^* \))

\[
L^* = 116 \min \left( \frac{Y}{Y_w} \right) - 16 \\
a^* = 500 \min \left( \frac{X}{X_w} - \frac{Y}{Y_w} \right) \\
b^* = 200 \min \left( \frac{Y}{Y_w} - \frac{Z}{Z_w} \right)
\]

with \( h(q) = \begin{cases} \frac{q}{0.008856} & q > 0.008856 \\ 7.787q & q \leq 0.008856 \end{cases} \)

- \( X_w, Y_w, \) and \( Z_w \) are reference white tristimuli values (D65 illumination)
- The Just Noticeable Difference (JND) in \( a^*b^* \)-space is 2.3
Device-dependent color models depend on the primaries of the hardware.
- RGB: cameras and scanners employ Red, Green and Blue spectral filters.
- RGB: monitors employ Red, Green and Blue emitting phosphors.
- CMY / CMYK: printers deposit Cyan, Magenta, Yellow and black colored pigments (colorants).
- HSI: Hue, Saturation (chroma) and Intensity are used to describe color, but are not directly related to the Human Visual System (HVS). HSI is an alternative representation for RGB and can hence be derived from it.

Perceptual color models use Color Matching Filters (CMF’s)
- CIE RGB: tristimulus values obtained by CIE RGB CMF’s
- CIE XYZ: tristimulus values obtained by CIE XYZ CMF’s

Perceptually uniform color models based on CIE XYZ
- CIE L*a*b*: Lightness and approximately uniform uv
- CIE L’*a’*b’*: Lightness and approximately uniform ab
Color recording systems

Aspects that are important for characterization are:
- Linearity of the individual color channels (RGB)
- Space invariance
- Light source

Color calibration converts the measured RGB responses to a perceptual color model.
- Use a test chart with color patches (known XYZ for calibrated light source, D65)
- Measure the RGB responses
- Determine the relation between RGB and XYZ color models

Color reproduction systems

Monitors
- Additive process: \( a \lambda(i) + b \lambda(i) + c \lambda(i) \)
- Emission spectra of the phosphors
- Linearity of irradiance: gamma correction

Inkjet printers
- Subtractive process:
  \[ s(\lambda) t_1(\lambda) t_2(\lambda) t_1(\lambda) t_2(\lambda) t_3(\lambda) \]
- \( I(\lambda) \): spectrum of the light source:
- \( t_i(\lambda) \): transmission spectrum of the colorants: \( t_i(\lambda) \)
- \( r(\lambda) \): Reflection spectrum of the paper: \( r(\lambda) \)
- Dithering schemes, half toning

Calibration charts
- ICC profiles
- Gamut mapping

Pseudo color

- Pseudo color is a common technique to replace the gray-scale by a color map.
- The color map does not have to bear a relation with a physical interpretation of the data.
- The color map can be selected in such a way that some “gray-tones” stand out.

Pseudo color for detection
Full color image processing

Full color image processing refers to manipulation and processing of color images.

We distinguish:
- Color manipulations: global point operations to change the contrast, color tone, white balance.
- Color image processing: local processing of color images to extract, enhance, or process otherwise to produce a color image as output.
- Color segmentation: partitioning of a color image into relevant regions based on similarities of color attributes.

Color manipulations

- White balance and color tones
  The color of the reflected light depends on the spectrum of the light source. Although the HVS is relatively insensitive to variations in illuminations, a color image sensor is not.
Color image processing

- Smoothing and sharpening
- Which color representation to choose
- Filtering all RGB channels vs filtering the Lightness component
- New colors (new hues) can occur
- What about the gamut?
- Suppression of noise in color images

Sharpening: 1-Laplace filter

- Processing of all RGB channels
- Processing of the Lightness channel

Blurring: Gaussian filter

- Processing of all RGB channels
- Processing of the Lightness channel

Color segmentation

- Color vectors (pixels) that belong to the same object should form clouds in color space.
- Problem: Non-uniform illumination yields very elongated tracks in RGB color space.
- Solution: Use a color model in which the intensity can easily be decoupled from the color information:
  - Hue + Saturation
  - CIE a + CIE b