

LOAD AND DEFORMATION ANALYSIS IN SOLID MECHANICS

by

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CHAPTER 1

BASIC CONCEPTS IN LOAD ANALYSIS

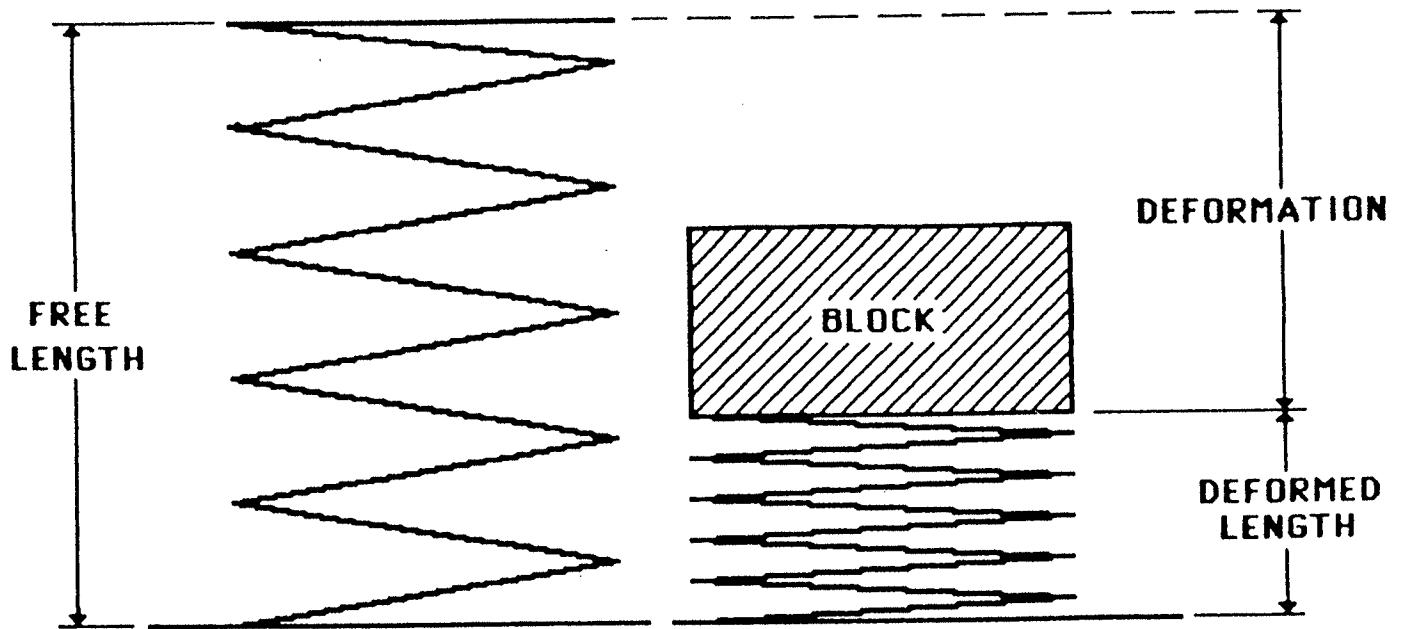
1.1 INTRODUCTION

In this text we discuss static load analyses for the members in a structure and the components in a machine, generically termed solid bodies. Static load analyses are based on static equilibrium conditions, where, by definition, a solid body originally at rest or moving with a constant velocity remains at rest or continues to move with the same velocity even after being acted upon by external loads. These external loads cause deformations that, although small, may be of critical importance. Accordingly, in design, we must consider both the loads acting on solid bodies and the deformations which invariably accompany these loads.

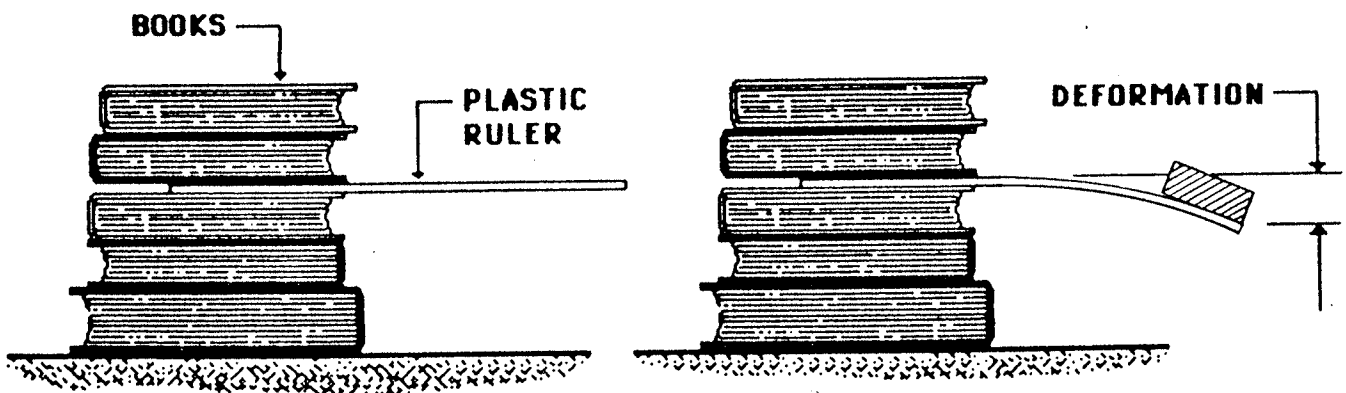
Our objective in load analysis is to determine the reaction loads which satisfy equilibrium conditions for a solid body that is acted upon by a specific set of applied loads. These reaction loads occur at the locations of contact between the solid body and its supports or connections with other solid bodies. However, we are severely limited in our ability to accomplish this objective because actual load transfer conditions are typically very complex. Thus we have no practical alternative but to approximate, as best as we can, actual load transfer conditions by "equivalent" idealized load transfer conditions. This approximation process is called load modeling. Although we seldom have the resources to assess by appropriate experimentation the accuracy of our load modeling approximations, we can often establish model adequacy by stating and evaluating

various modeling alternatives in a methodology called design load analysis. It is essential to understand at the outset that because of the idealizations involved in design load analyses, the solutions obtained cannot be considered exact; they are at best only adequate for practical purposes.

In this first chapter, we introduce certain fundamental concepts and definitions useful in load analyses. Our emphasis is placed on physical understanding rather than on mathematical details.



a) BLOCK PLACED ON A "SOFT" COIL SPRING



(b) BLOCK PLACED AT THE END OF A PLASTIC RULER.

FIG. 1.1: EXAMPLES OF VISIBLE DEFORMATION

1.2 LOAD CLASSIFICATION

A load applied to a solid body causes or tends to cause movement; specifically either a translation, or a rotation, or both. This movement provides a convenient means to classify or define certain external loads. If an applied load causes or tends to cause only translation, it is a force. If the applied load causes or tends to cause only rotation, it is a couple.

External loads, no matter how small, cause deformations. In some cases, the deformation is very small and not readily discernible. In other cases, the deformation is easily recognized by a change in the geometry (dimensions) of the solid body. If a small steel block is placed on a concrete floor, the deformation of the floor, localized only in the area of contact with the block, is so minute that it cannot be detected without resorting to very sensitive instrumentation. However, if the same block is placed on a "soft" coil spring, or at the end of a flexible plastic ruler, there is an observable deformation in each case. The spring decreases in height whereas the straight plastic ruler becomes curved by bending, Fig. 1.1.

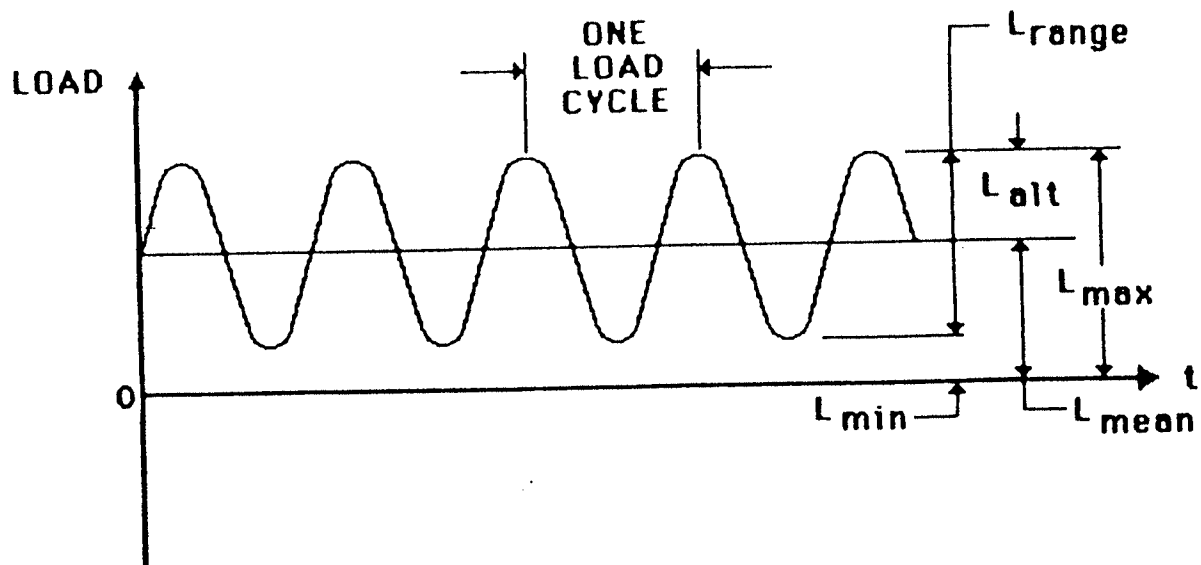
1.2.1 Static and Dynamic Loads

Suppose a solid body acted upon by external loads either remains at rest or moves with a constant translational velocity. Since the body is in static equilibrium, we refer to the external loads as static loads. On the other hand, if the solid body experiences an acceleration or a deceleration, caused either by a change in the magnitude or the direction of the translational velocity, we refer to the external loads as dynamic loads. Extreme cases of dynamic loads are termed impact loads. When an automobile is driven along a straight highway at a constant speed, its driver and certain of its components such as bumpers, headlights, hood, etc., experience static loads. However, components such as connecting rods, pistons, intake and exhaust valves, etc., experience dynamic loads. When the automobile speeds up or slows down, its driver and all of its components experience dynamic loads. If the automobile hits a solid barrier, it experiences an impact load condition.

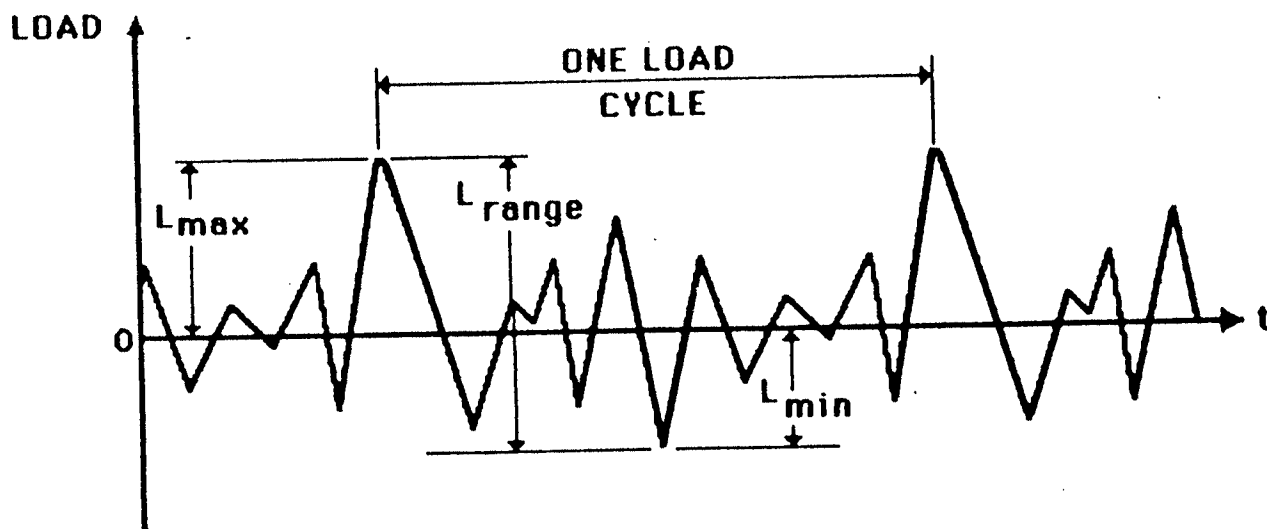
External loads acting on solid bodies are considered static loads when their dynamic effects are negligible for practical purposes. Static loads are not necessarily time invariant, only their effect on the motion of a solid body is time invariant for practical purposes. If a static load remains unchanged during the entire time of its application, as in the case for the weight of a structure, we call it a dead load. If a static load remains unchanged only over some portion of the total operational cycle of a device, machine, or system, we call it a steady load. On the other hand, cyclic loads repeat their time histories periodically during the service operation of a device or a machine. Examples of cyclic loads are shown in Fig. 1.2. Cyclic loads may be either static or dynamic, depending on whether

their dynamic effects are negligible for practical purposes.

A dynamic load analysis must be performed to determine whether, in a given situation, the dynamic effects are in fact negligible for practical purposes. This issue reflects a major weakness of static load analysis. But all analyses have weaknesses that must be evaluated critically before interpreting and using their results. Moreover, all analyses can be improved as knowledge and information accumulate. Thus, our static load analyses should subsequently be modified to consider dynamic effects (when the student has attained the appropriate background in subsequent course work). This modification may reasonably be viewed as merely another step in a continuing iterative design process.



(a) Simple cyclic load.



(b) More complex cyclic load.

FIG. 1.2

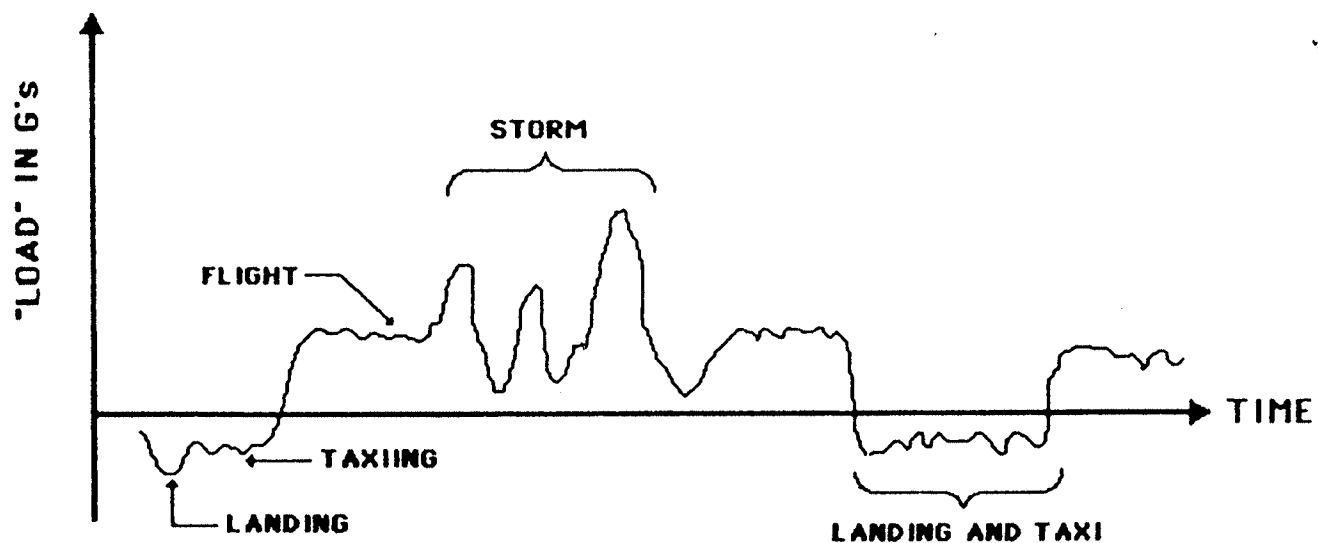


FIG. 1.3: Service load history for the main spar fitting of a transport aircraft. Note that the load is stated in G's, where one G is equal to the weight of the aircraft.

1.2.2 Service Load Histories

Load histories associated with various components in a machine or a structure during service operation are seldom as simple as depicted in Fig. 1.2. Rather they are usually very complex. Consider, for example, the load acting on the main spar fitting which attaches the wing to the fuselage of a transport aircraft. The service load history from take-off to landing is illustrated schematically in Fig. 1.3. This load history is complex because it depends on factors such as air turbulence, maneuvers, amount of fuel and fuel tank location in the wing, etc. Each of these factors can vary markedly during a flight as well as among "similar" flights. Thus, the service load history in this example cannot be described adequately without using statistical considerations.

In most engineering applications component loads can be determined only by measuring them during some phase of actual service operation. Since component load history measurement requires sophisticated instrumentation as well as substantial effort, it is common design practice to circumvent measurement and instead to employ "engineering experience and judgment" in selecting some simple load history to represent a relatively extreme service operation condition. The component is then tested under this extreme load history and if it passes this design bogey test, it is said to have met or exceeded the design objective.

An alternative design approach, useful in some cases, is to estimate the lower bound of service loads by considering those loads associated with the steady state operation of a machine. The transient loads which occur during start-up, shut-down, and sudden accelerations or decelerations of the machine may be much

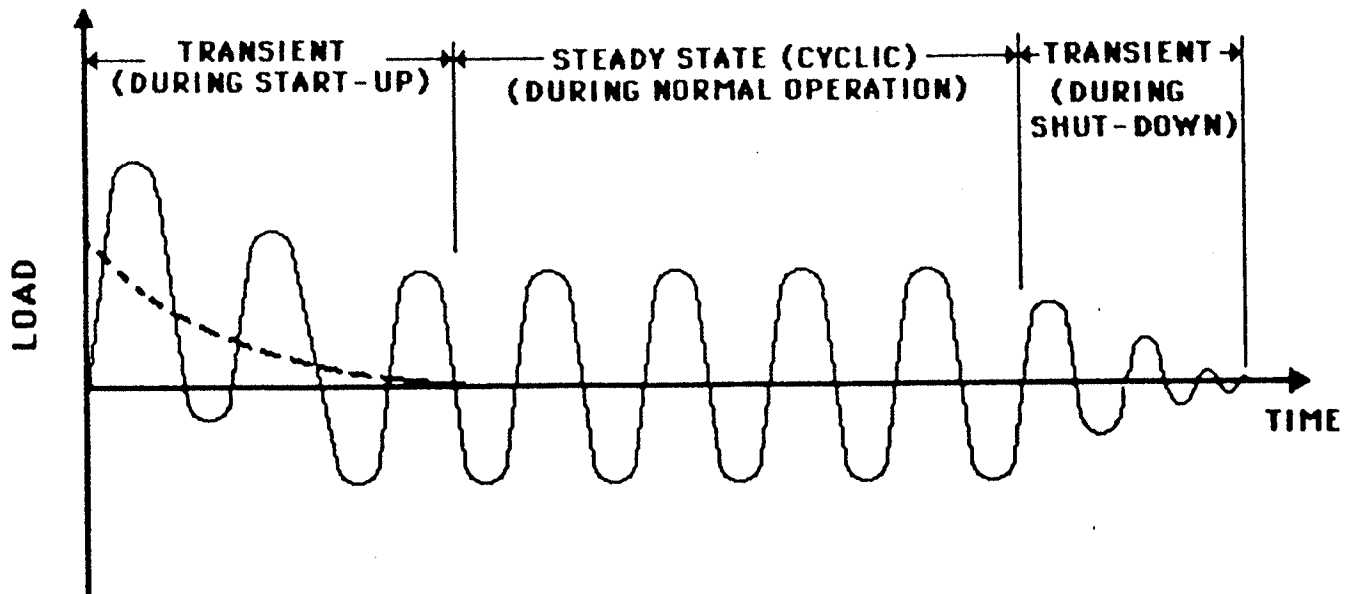


FIG. 1.4: Steady state and transient loads in a machine.

higher than the steady state loads; however, they appear only over short periods of time during the operation of the machine, Fig. 1.4. An estimate of the maximum value of the transient loads can be made by multiplying the steady state loads by a factor greater than unity. The size of this "dynamic" factor is a matter of engineering experience and judgment, but it usually falls in the range of 1.25 to 2.5.

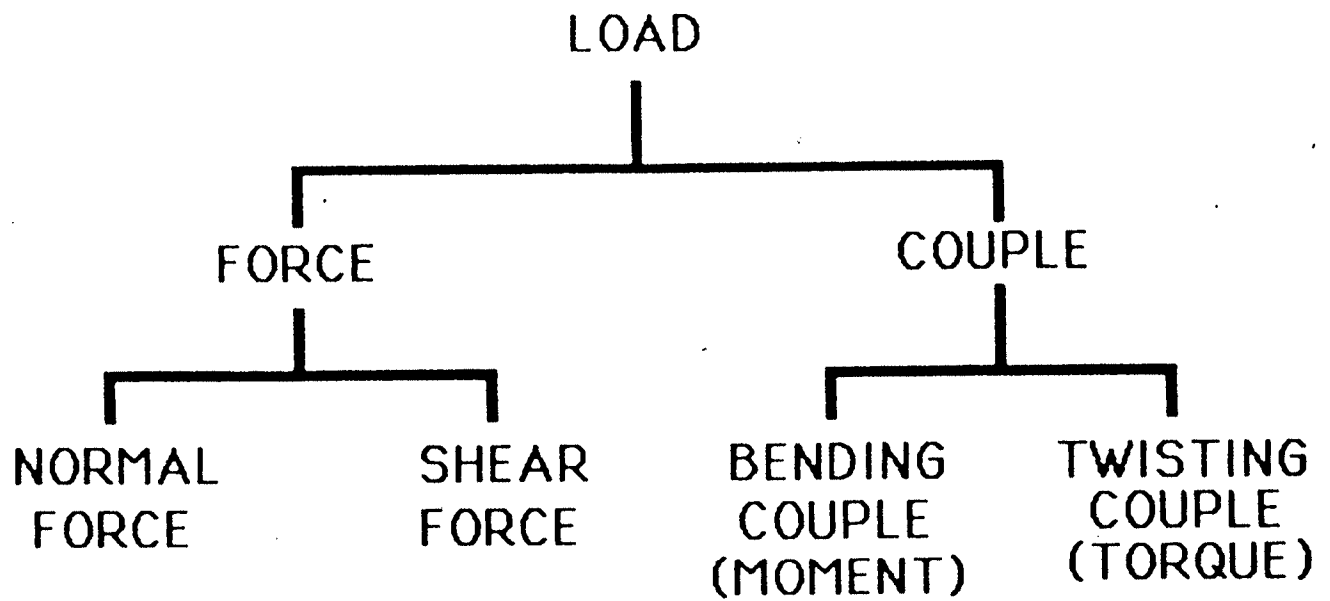


FIG. 1.5 Load Classification according to deformation

1.2.3 Load Classification Based on Deformation

Loads are often classified on the basis of the type of deformation produced, Fig. 1.5.

A normal force acts normal (perpendicular) to the surface or cross-section of a solid body, Fig. 1.6(a) and (b). It causes a normal deformation, i.e., an element is either elongated or a shortened in the direction normal to the surface, depending on whether the normal force is tensile or compressive.

A shear force acts parallel to the surface or cross-section of a solid body, Fig. 1.6(c). It causes a shear deformation, i.e., a square element becomes a parallelogram.

A bending couple acts perpendicular to the longitudinal axis of the solid body. It produces non-uniform normal deformations, Fig. 1.6(d), such that the solid body bends with a curvature in the plane perpendicular to the applied couple.

A twisting couple, also called a torque, acts parallel to the longitudinal axis of the solid body. It produces shear deformations (torsion) such that the solid body twists with a rotation about its longitudinal axis Fig. 1.6(e).

The deformations produced by each of these loads can be demonstrated using a soft rectangular rubber eraser (Fig. 1.7). Draw several small square grids on the top, bottom and sides of the eraser and then apply loads with your fingers so that the eraser is stretched, compressed, sheared, bent and twisted. When an eraser grid element is stretched, the square element becomes rectangular, elongated in the direction of the tensile force (pull), Fig. 1.7(a). When an eraser grid element experiences a shearing deformation, the square element becomes a parallelogram, Fig. 1.7(b). Bending makes the eraser curved such that square grid elements on

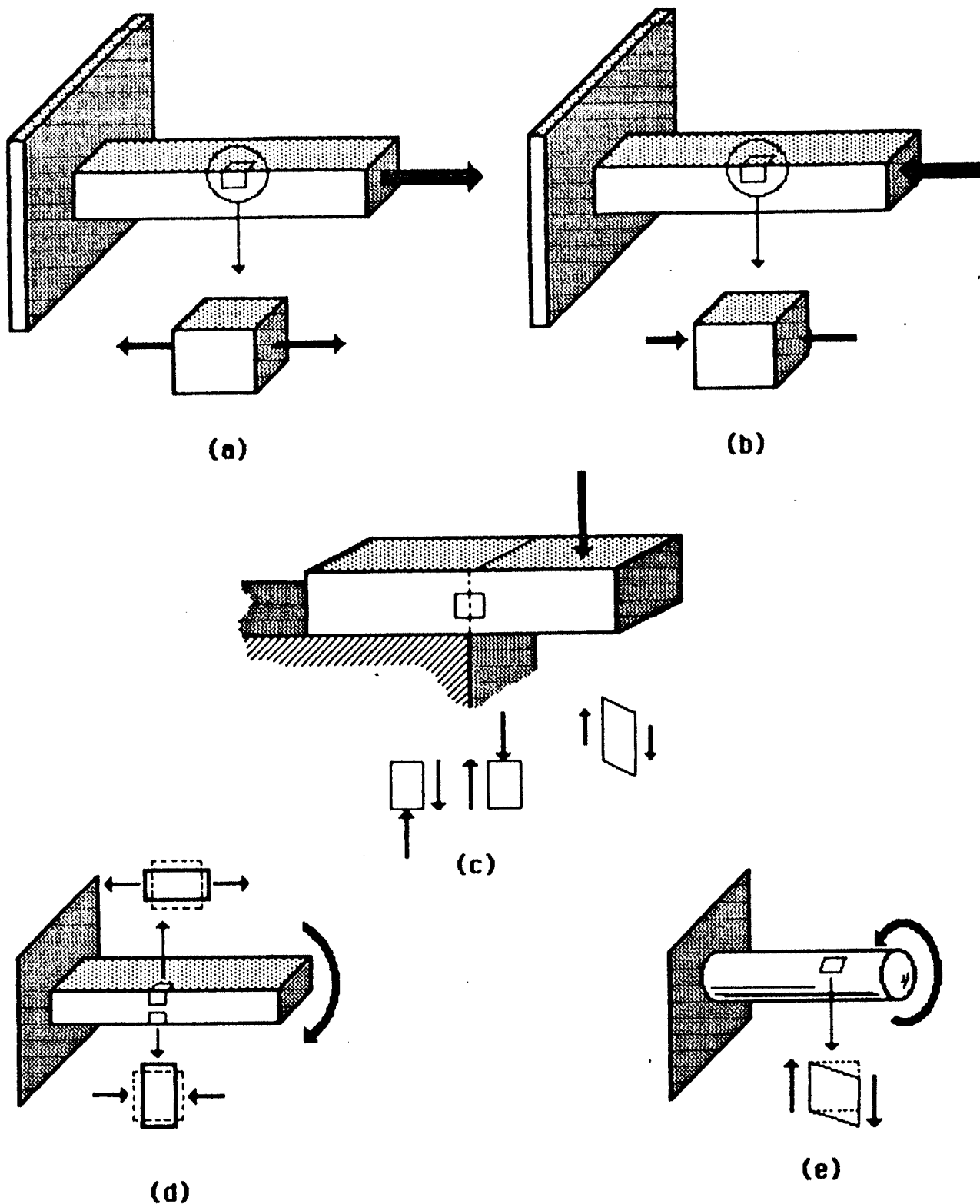
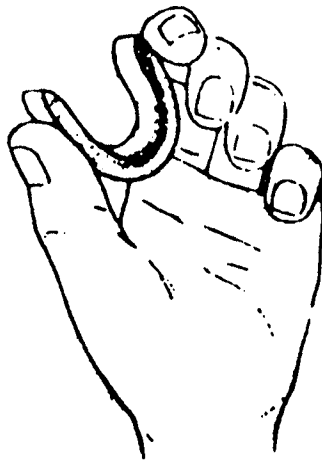
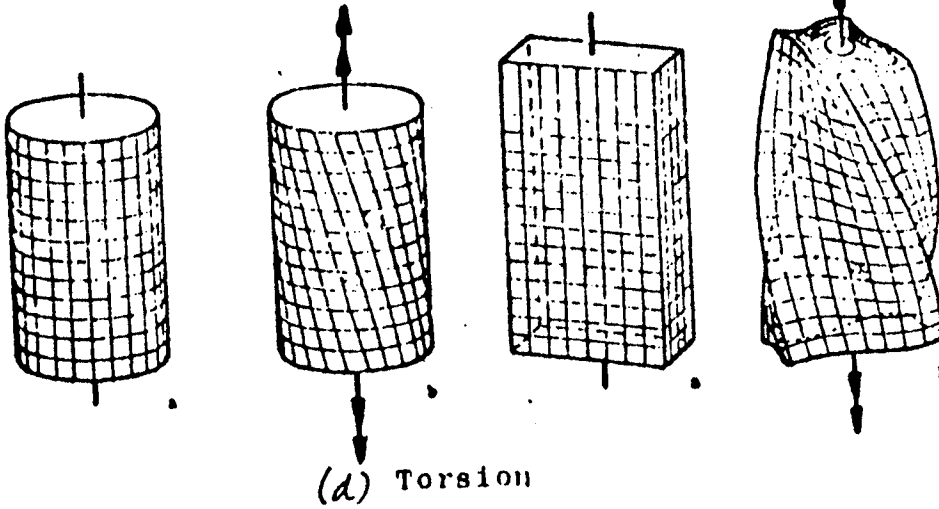


FIG. 1.6: Example of deformation associated with (a) normal tensile forces, (b) normal compressive forces, (c) shear forces, (d) bending couples, and (e) twisting couples (torque).



(c) Bending



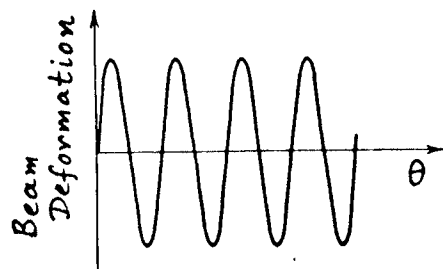
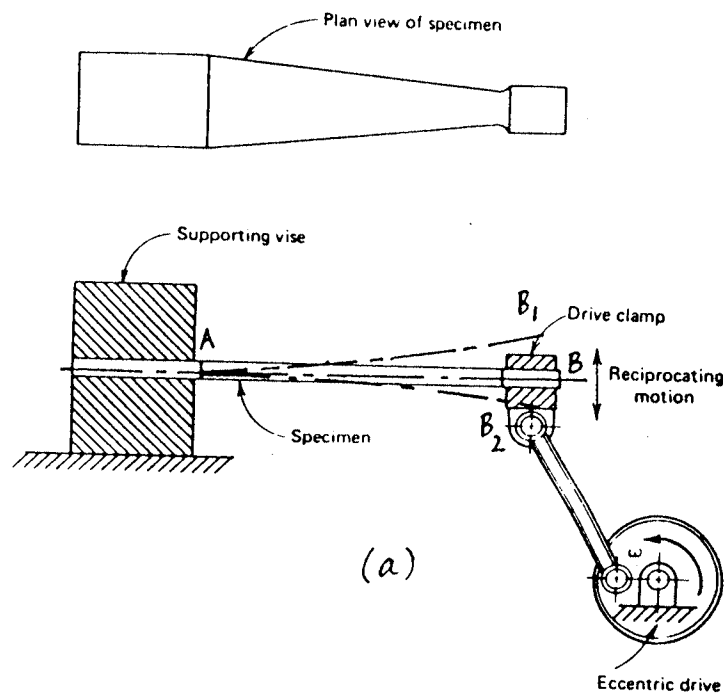
(d) Torsion

Fig. 1.7

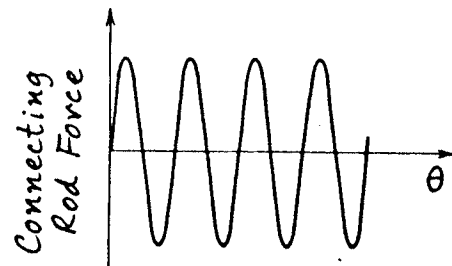
the top (bottom) become rectangular by elongation, whereas square grid elements on the bottom (top) become rectangular by compression, Fig. 1.7(c). If the eraser has a rectangular cross-section, the shear deformation of its grid elements depend on their location, i.e., different grid elements deform by different amounts, Fig. 1.7(d). On the other hand, if the eraser has a circular cross-section, then it would be seen that each grid element exhibits the same shear deformation.

Observe in performing the eraser experiment that it takes much less physical effort to obtain large deformations by bending or twisting than by pushing, pulling or shearing. Note also that the deformed grid elements return to their original shape and size when the external load is removed. All of the deformations observed in this eraser experiment are much larger than those occur in the vast majority of practical applications.

Although deformations always accompany applied loads, it is usually assumed in load analysis that the solid body behaves in a "rigid-like" manner. In other words, it is usually assumed that, regardless of the magnitudes of the applied loads, the geometries of the undeformed and deformed solid body are identical for practical purposes. Obviously, this assumption is credible when the deformations caused by the applied loads are very small.



(b)



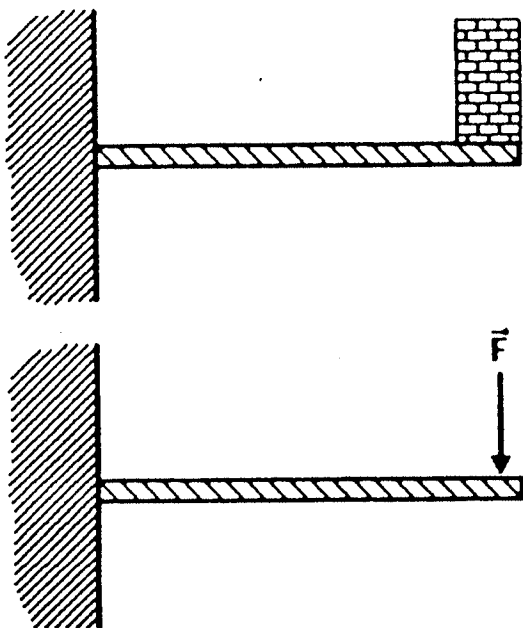
(c)

Fig. E 1.1 : Plate Bending Fatigue Machine

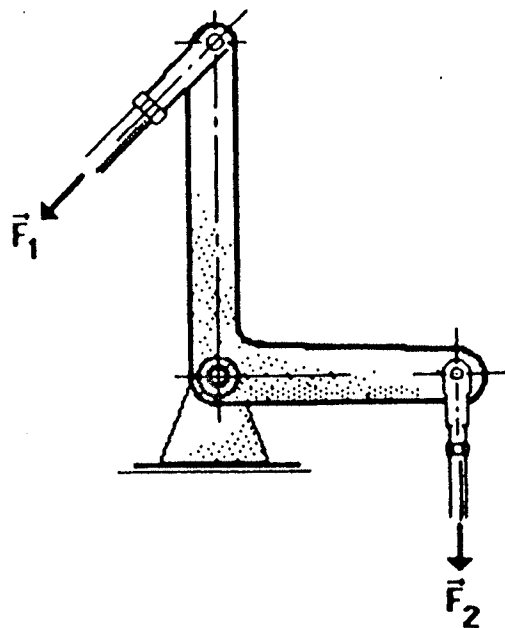
Example E1.1: Load-Deformation-Time Relationships

Fig. E1.1(a) shows the schematic of a laboratory fatigue (plate bending) testing machine in which a long, slender specimen is clamped at one end (A) and flexed at the other end (B) by means of an eccentrically rotating crank. As the crank rotates, the specimen bends upward from its unflexed position AB. As the specimen is flexed upward, its top surface is compressed, whereas its bottom surface is stretched (compare this with the eraser experiment). As the crank rotates through another 180 degrees, the specimen is first brought back from its extreme up position AB_1 , to the unflexed position AB and then is flexed downward to position AB_2 . In this latter position, the top surface of the specimen is stretched, while the bottom surface is compressed. Fig. E1.1(b) displays this time dependent variation of deformation as a function of the crank angle, θ , in Fig. E1.1(b).

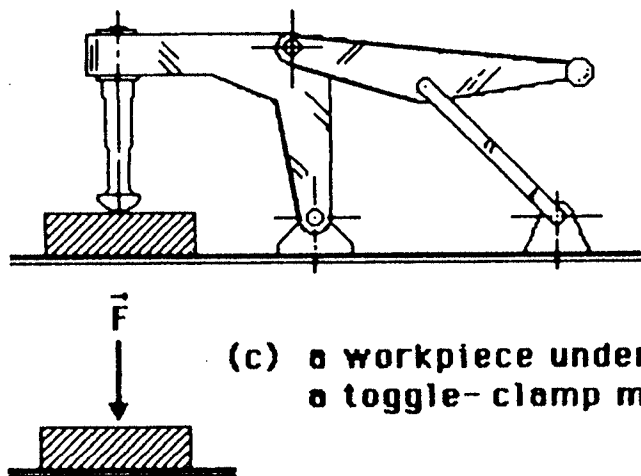
In this fatigue example, the primary mode of specimen deformation is bending caused by the rotation of the crank. The connecting rod force which acts at the end of the specimen and causes the bending deformation also varies cyclically as a function of the crank angle, θ , as shown in Fig. E1.1(c). Since the crank rotates at a constant angular velocity during the fatigue test (which may continue for several days or even several weeks), the associated variation of the connecting rod force can also be expressed as a cyclic function of time.



(a) bricks stacked at the end of a cantilever platform.



(b) rod and yoke (clevis) connections acting on a bell crank mechanism.



(c) a workpiece under the stylus of a toggle-clamp mechanism.

FIG. 1.8

1.2.4 Classification of Load (Force) Based on Area of Application

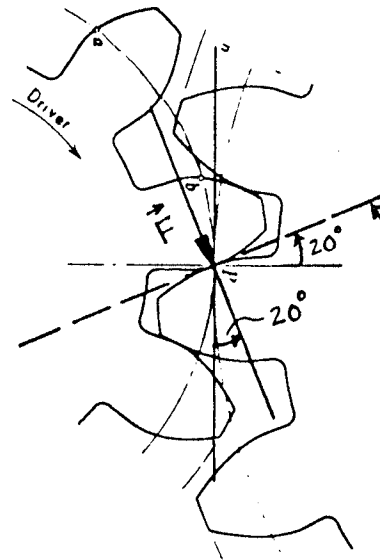
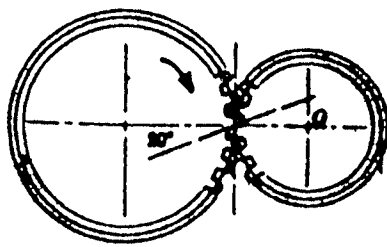
Load transmission occurs primarily by physical contact between two solid bodies. In this context, all loads are actually distributed forces, i.e., distributed over the area of contact. However, in load modeling we classify loads according to the size of the "apparent" area of contact.

If the apparent area of contact is very small compared to important dimensions of the bodies, we model the distributed force as a

concentrated force and represent it by a (single) force vector, Fig.

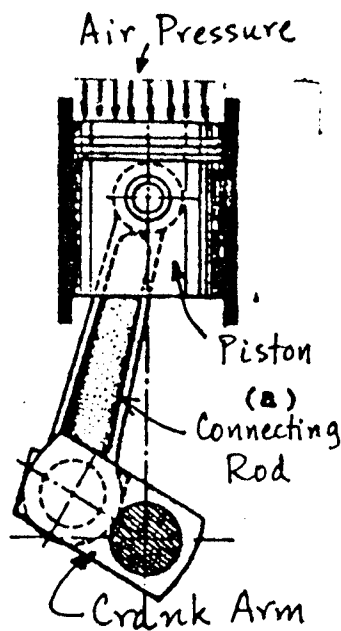
1.8. A distributed force, in contrast, is applied over a relatively large area of the body. Distributed forces are represented by a series of parallel force vectors whose relative lengths indicate the distribution of the force per unit area. If one of the dimensions of this area is small compared to the other, we idealize the distributed force as acting along a line. The force distribution, in the case of line contact, is given by force per unit length. The set of two spur gears in Fig. 1.9 provides an example of line contact.

Distributed forces occur in various ways. For example, the air pressure in Fig. 1.10(a) is assumed to be uniformly distributed over the cross-sectional area of the piston. Water pressure acting on the wall of a dam is non-uniformly distributed, since its magnitude is directly proportional to depth, Fig. 1.10(b). In most applications, however, the force distribution is complex and cannot be easily measured, e.g., the force distribution acting over the "foot print" of a rolling automobile tire, Fig. 1.10(c).



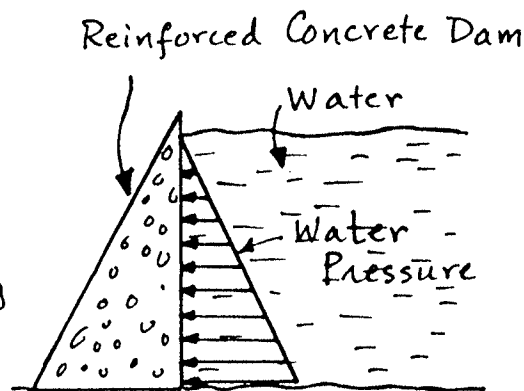
Common tangent to the curved surfaces of gear teeth in contact.

Fig. 1.9



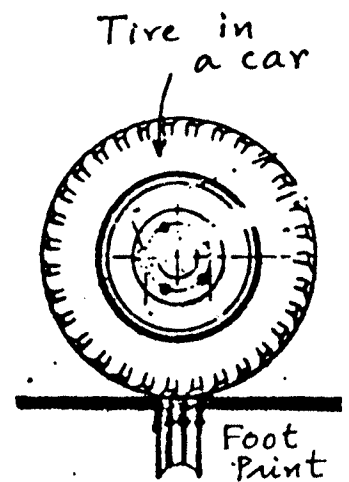
Uniformly Distributed Load (known)

(a)



Non-uniformly Distributed Load (known)

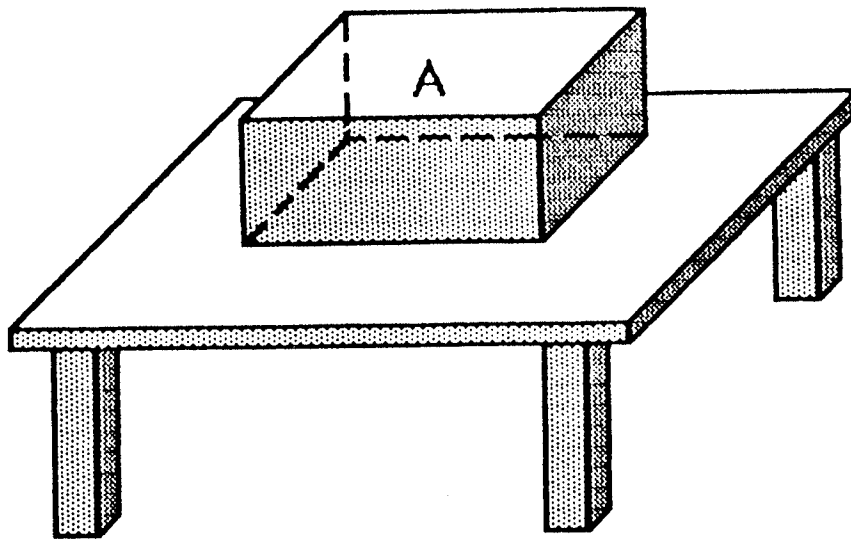
(b)



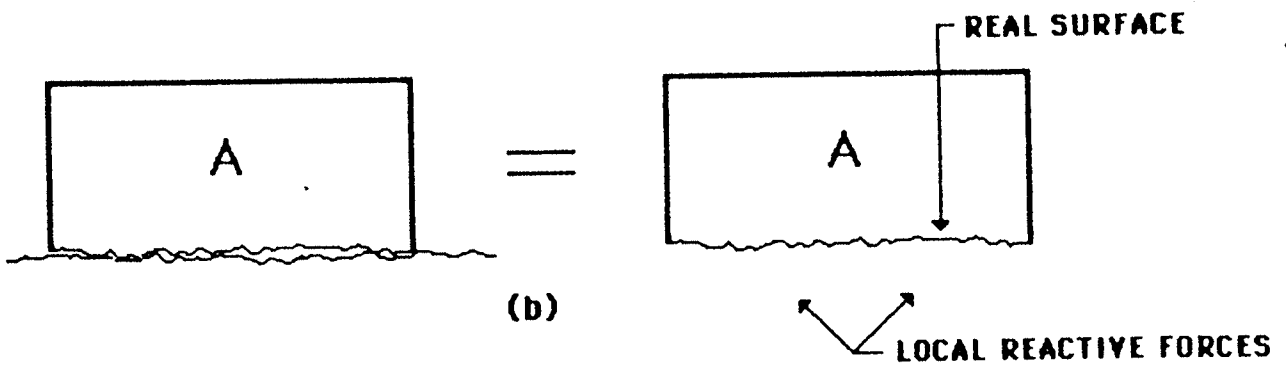
Non-uniformly Distributed Load (actual distribution unknown)

(c)

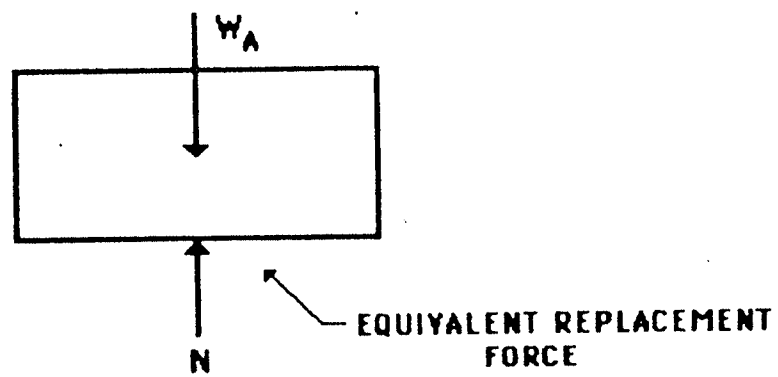
Fig. 1.10



(a)



(b)



(c)

FIG. 1.11

1.3 LOAD MODELING

A force cannot physically act at a point or be distributed along a line. Such force representations are conceptual "equivalents" of the actual loads. The mental process involved in establishing conceptual equivalents is called load modeling.

Consider a small metal block, A, resting on a horizontal table top, Fig. 1.11. The vertically downward action of gravity acting on the block is resisted by the reactive forces generated by the table surface. These reactive forces must be distributed in some manner over the contact area. The exact nature of the force distribution is not known, however, due to the fact that surfaces of the block and the table top, or for that matter, the surfaces of any two solid bodies, are never perfectly smooth or flat. The roughness of a surface can usually be reduced by improved manufacturing techniques; however, it can never be completely eliminated. Due to the inherent roughness of all surfaces, local contact always takes place only at the high spots on a microscopic level (called asperities), as shown schematically in Fig. 1.11(b). Thus the actual area of contact is much smaller than the conceptual (apparent) area of contact. Since the roughness of each surface can only be described statistically, the magnitudes and directions of the local reactive forces cannot be stated with certainty. Therefore, we model both the actual surface by an "idealized" smooth flat surface and the numerous local forces by a (single) force vector whose physical action is "equivalent" to the action of the (numerous) local forces. This equivalent force must prevent the vertically downward motion of the block. Accordingly, it must act vertically upward, normal (perpendicular) to the conceptual smooth flat surfaces and is therefore a "normal" force. The magnitude and location of this concentrated normal force is determined by static equilibrium

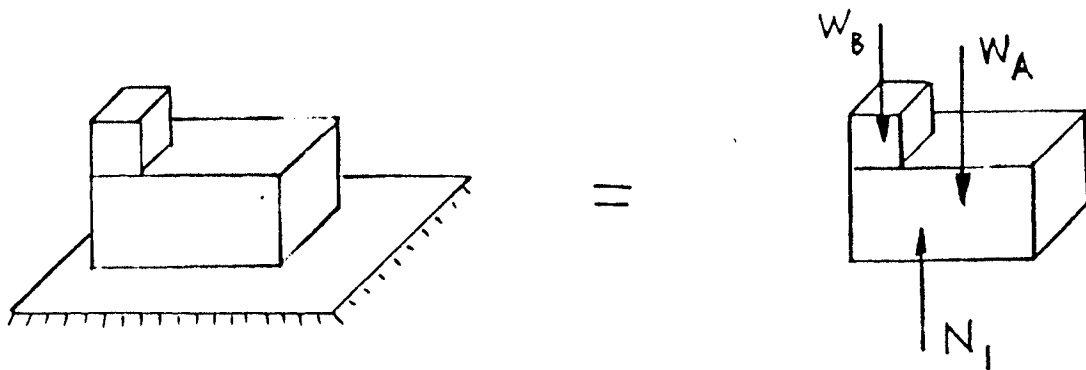


Fig. 1.12

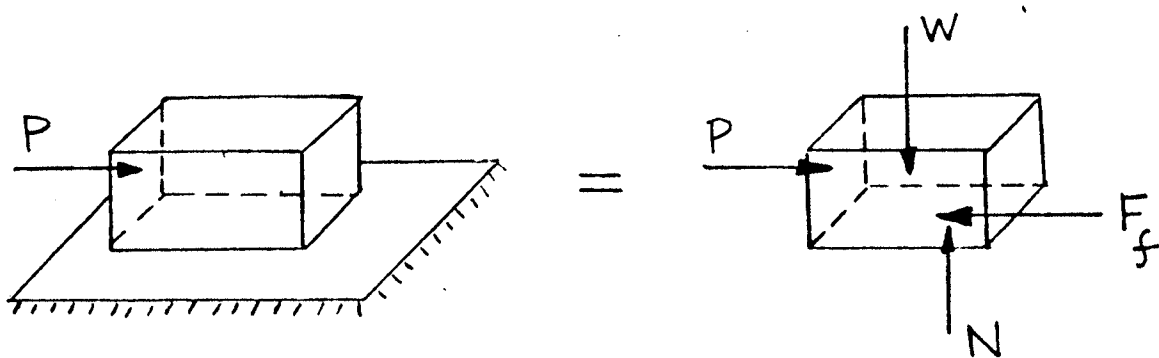


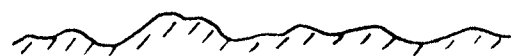
Fig. 1.13

analysis. But this example is so simple that we know by intuition that the normal reaction force, N , of the table top acting on the block is equal in magnitude, opposite in direction and collinear with the gravitational force, W , the weight of block A, Fig. 1.11(c). It is also intuitive that the horizontal components of the local reactive forces must sum to zero, or else the block would slide horizontally.

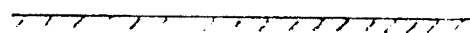
Suppose, in turn, we place a smaller block B on top of the original block A, Fig. 1.12(a). How does this change the actual nature of contact and the resulting reactive forces of the desk top acting on block A? Here, again, it is impossible to specify the exact nature of the (numerous) local reactive forces. Therefore, we again model the action of the table top on block A using a concentrated normal force, N_1 , which has a physical effect on block A that is "equivalent" to the effect of the local reactive forces, Fig. 1.12(b). Obviously, N_1 will differ from N , both in terms of magnitude and location.

If a horizontal force P is applied to block A, as shown in Fig. 1.13(a), the block will have a tendency to slide along the table top to the right. However, we know that if the applied force is sufficiently small, the block will not slide. If it does not move, the local contact must change such that the reactive forces produced exactly balance the applied horizontal force P and thereby prevent the block from sliding. These reactive forces occur only at localized areas of contact (asperities) where mechanical interlocking and metallurgical bonding (welding) take place on a microscopic level. We model the equivalent of these reactive forces as a (single) concentrated force which creates the same physical effect on the block as the actual reactive forces. We call this specific equivalent force a (concentrated) "friction force", F_f , Fig. 1.13(b) and determine its magnitude and location by static equilibrium analysis.

The direction of the friction force is usually determined by considering the direction of the relative motion of one surface with respect to the other. Friction forces act to oppose actual or impending sliding between the contacting surfaces.



Actual Surface
(Rough)

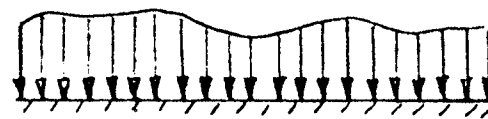


Idealized Surface
(Smooth)

(a)

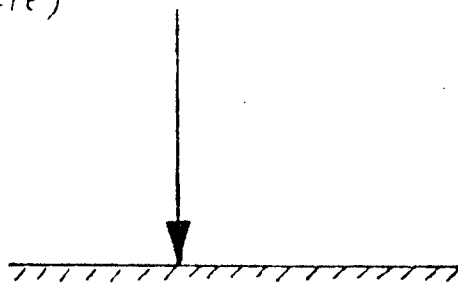


Actual Forces of Contact
(Discrete)



Idealized Forces of Contact
(Distributed)

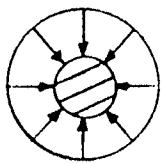
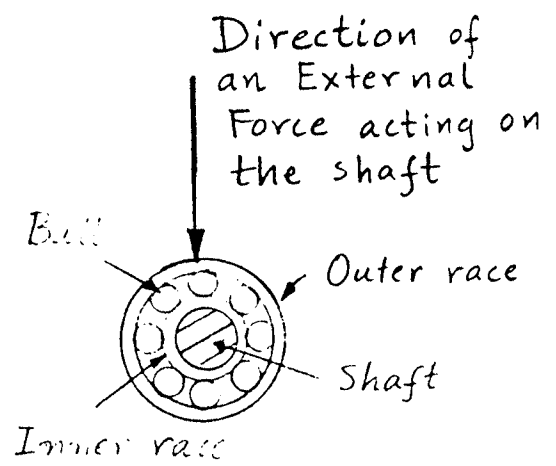
(b)



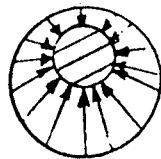
Equivalent Force
of Contact

(c)

Fig. 1.14



(a)



(b)



(c)

=



(d)

Fig. E 1.2

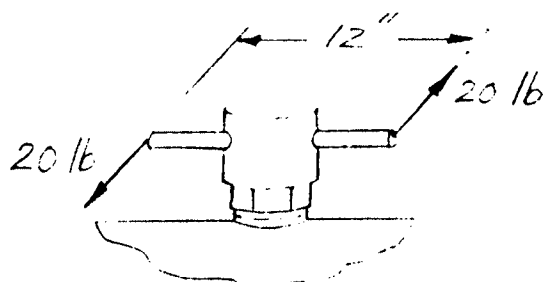
1.3.1 Generalized Modeling Procedure

The physical nature of the actual local contact is so complicated that we must adopt a simplified model. First, we ignore the actual surface roughness and visualize instead on the conceptual (idealized) smooth, flat surface, Fig. 1.14(a). Next, we ignore the actual contact forces and visualize instead the equivalent conceptual (idealized) force distribution. Thus, we model actual contact using both a conceptual surface and a conceptual (continuous) force distribution, Fig. 1.14(b). In turn, we usually simplify the modeling even further by replacing the conceptual force distribution by an equivalent (conceptual) concentrated force, Fig. 1.14(c). These simplifications allow us to state the essential aspects of the physics of the problem using simple mathematical expressions.

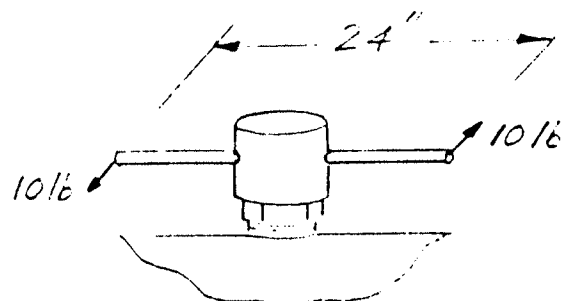
Example E1.2: Load Modeling (Two-dimensional)

Consider preliminary load modeling for a deep-groove (Conrad) ball bearing, Fig. E1.2. First, the light press-fit of the inner race of the bearing on the shaft ideally generates a uniform normal force distribution around the shaft circumference, Fig. E1.2(a). The external load applied to the shaft in turn changes this idealized force distribution. We intuitively expect that if the applied force acts vertically downward, the distributed normal forces on the inner race of the bearing acting on the shaft must increase at the bottom, with a corresponding decrease at the top, Fig. E1.2(b). However, the actual distribution is even more complex, because the normal force distribution must increase locally where the inner race is more rigid, namely directly over each ball, Fig. E1.2(c).

Regardless of the complexity of the "actual" distribution of normal forces associated with even idealized contact between the inner race of the bearing and the shaft, we know that the purpose of the bearing is to support the shaft so that it does not move vertically downward under the action of applied force. Hence we model the bearing reaction force on the shaft using a (single) concentrated force vector, Fig. E1.2(d), which has the same physical effect on the shaft as the "actual" distributed force. We determine the magnitude and direction of the concentrated force by using static equilibrium analysis.

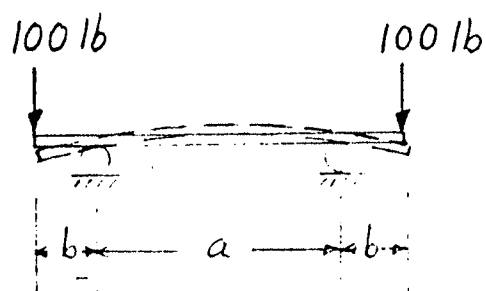


(a)

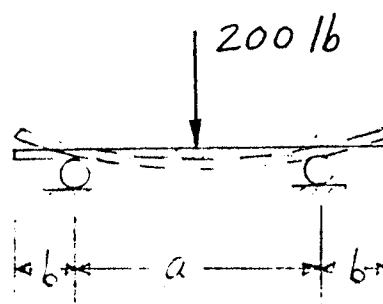


(b)

Fig. 1.15



(a)



(b)

Fig. 1.16

1.4 STATICALLY EQUIVALENT LOADS

Load modeling involves replacing the numerous forces associated with local contact by statically equivalent conceptual forces (loads). Two or more sets of loads are statically equivalent if their translational as well as rotational effects on a given solid body are identical. For example, two 20-lb forces applied at the ends of a 12-inch long lug wrench are statically equivalent to two 10-lb forces applied at the ends of a 24-inch long lug wrench, Figure 1.15.

Although statically equivalent loads have identical external effects on a solid body, they do not create the same internal loads and deformations. For example, two 100-lb forces applied at the ends of the overhanging roller-supported platform in Fig. 1.16(a) have an identical external effect as one 200-lb force applied at the center. Namely, both load sets produce the same 100-lb reactive forces at each support. However the associated platform deformations are completely different, as shown schematically in Fig. 1.16(b), and so are the respective internal loads.

The concept of statically equivalent loads is useful in modeling the external loads acting on a body, particularly when these loads are too complex to permit precise specification. However, this external load modeling technique can lead to serious errors if it is subsequently used in internal load analyses. Thus, subsequent use determines whether a given load modeling is competent or not. This issue will be discussed further in Chapter 6.

1.5 EQUILIBRIUM: NEWTON'S FIRST AND SECOND LAW

Newton's First and Second Laws state the equilibrium conditions for a particle:

First Law: A particle, originally at rest or moving with a constant velocity, will continue to remain at rest or move with a constant velocity in a straight line unless it is acted upon by an (unbalanced) force.

Second Law: The time rate change of velocity of a particle is directly proportional to and in the direction of the (unbalanced) force.

The Second Law reinforces the conditions of static equilibrium set forth in the First Law. For static equilibrium, the magnitude and direction of the velocity vector are time invariant; therefore its time rate change is zero, and so is the (unbalanced) force acting on the particle.

A particle is capable of translation only, whereas a solid body can undergo rotation as well as translation. Therefore, the conditions of equilibrium for a solid body involve both translational and rotational effects associated with external loads. Namely, a solid body is in the state of static equilibrium if and only if there are no unbalanced forces or couples acting on it. We will state this extension of Newton's First and Second Laws in equation form in Chapter 2.

1.6 ACTION AND REACTION: NEWTON'S THIRD LAW

Newton's Third Law states that for every action, there is an equal and opposite reaction. Thus, when two solid bodies are in contact with each other, the action of the first body on the second is resisted by an equal, but opposite reaction from the second body on the first, i.e., each has the same line of action but opposite senses. This principle of load transmission from one solid body to another is demonstrated in the following examples.

Example E 1.3: A block of weight W resting on a concrete floor.

Example E 1.4: Tension test of a flat specimen using pin grips.

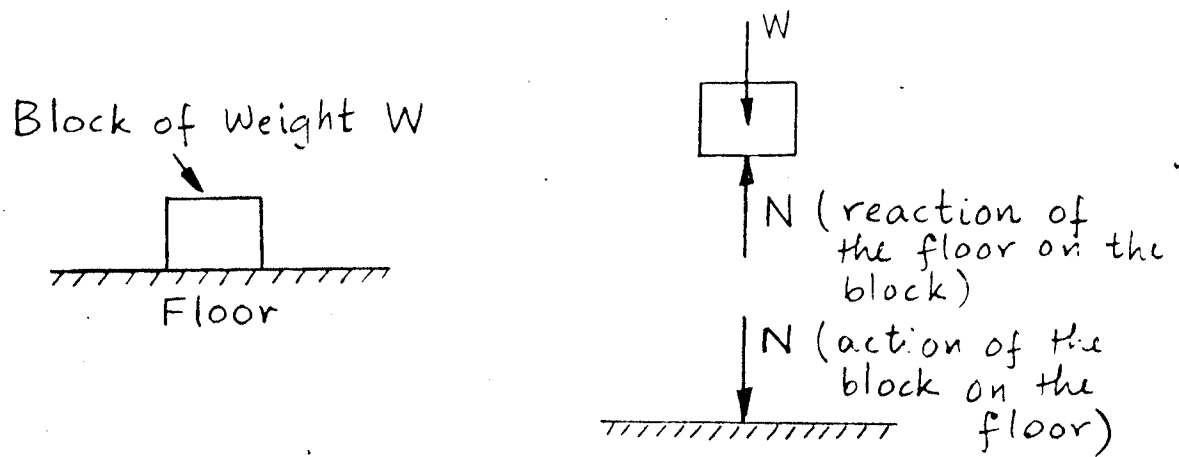


Fig E 1.3

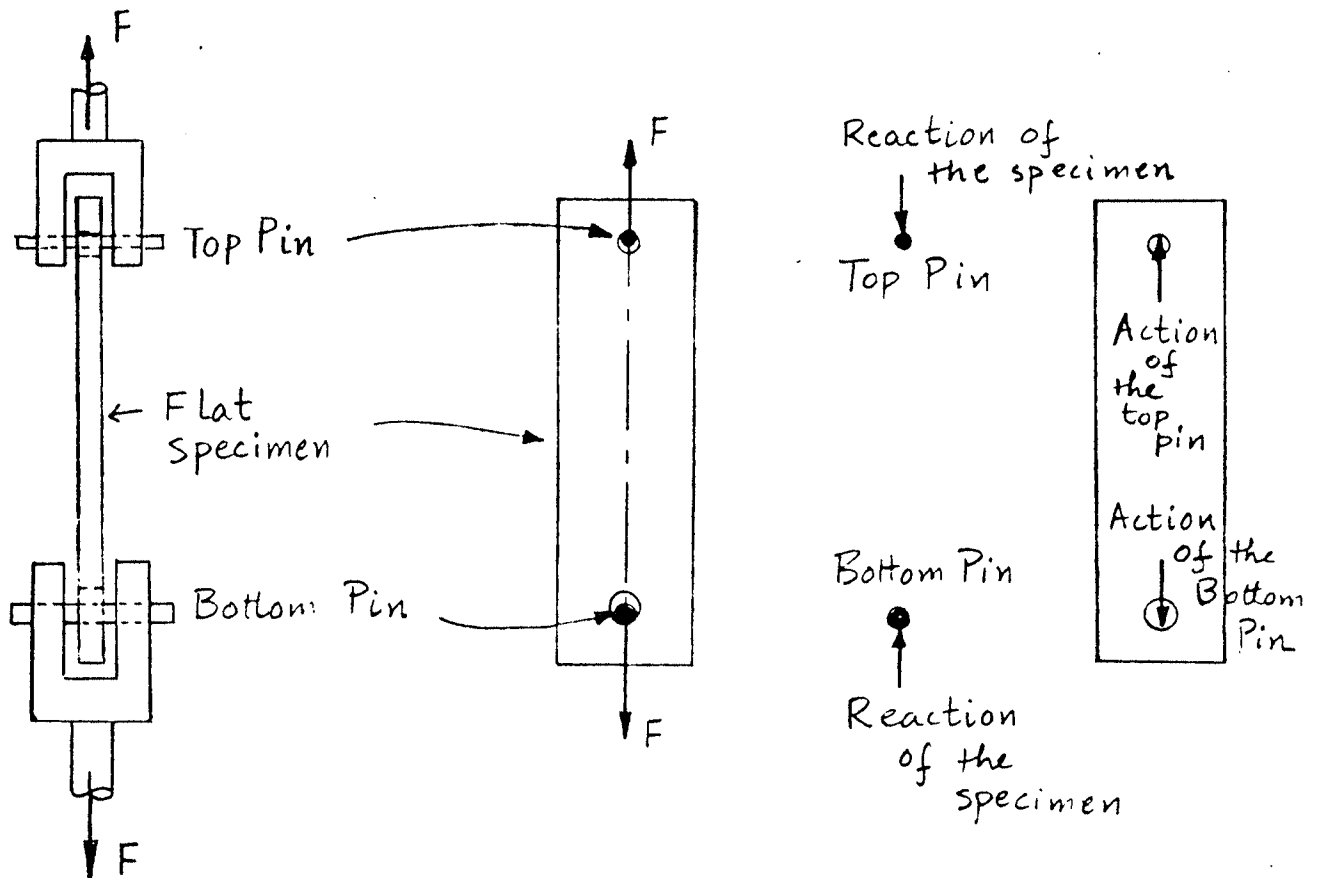


Fig. E 1.4

In the tension test using a pin-loaded specimen, a pin is inserted through the hole at the top and bottom of the specimen. As the grips holding the pins move apart, the pins make contact with the bearing surfaces of the pin holes in the specimen. By the principle of action and reaction, a force is transmitted from the grip to the pin to the specimen. The action of the pin at each end of the specimen is modeled by normal force which acts perpendicular to the common tangent to the respective curved surfaces at the "point" of contact.

Example E 1.5: A pair of meshed spur gears transmitting power from an electric motor to an output (drive) shaft.

The spur gear pinion is rotated in the clockwise direction at N_1 rpm (a) by the input shaft. Therefore, the twisting couple or torque exerted by the input shaft on the pinion is clockwise. The driven gear is rotated by the pinion in the counterclockwise direction at N_2 rpm so that

$$N_2 = \frac{D_1}{D_2} N_1$$

where, D_1 = pitch circle diameter of the pinion

D_2 = pitch circle diameter of the driven gear.

The torque transmitted by the driven gear to the output shaft is counterclockwise since it rotates in the counterclockwise direction. However, the reactive torque exerted by the output shaft on the driven gear is clockwise. In other words, the driven gear rotates in the counterclockwise direction against the clockwise output torque.

The forces acting on the pinion are the force exerted by the input

(a) rpm stands for revolutions per minute.

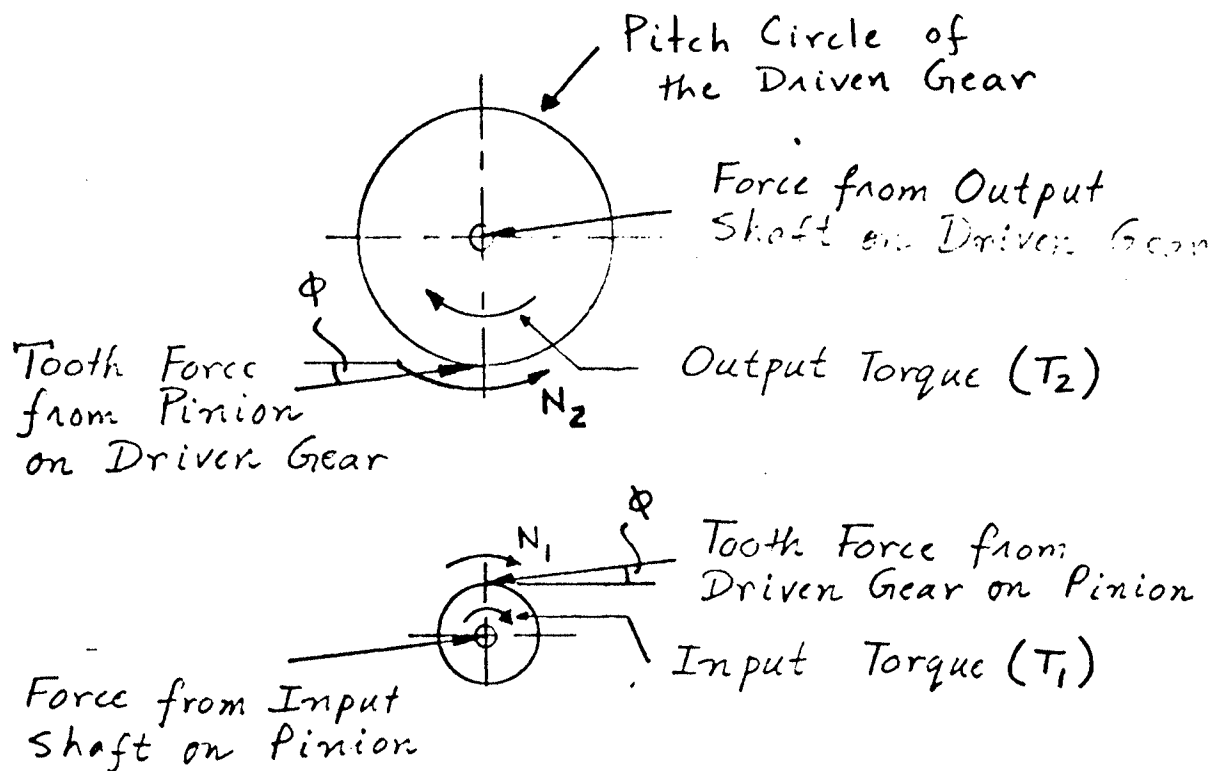
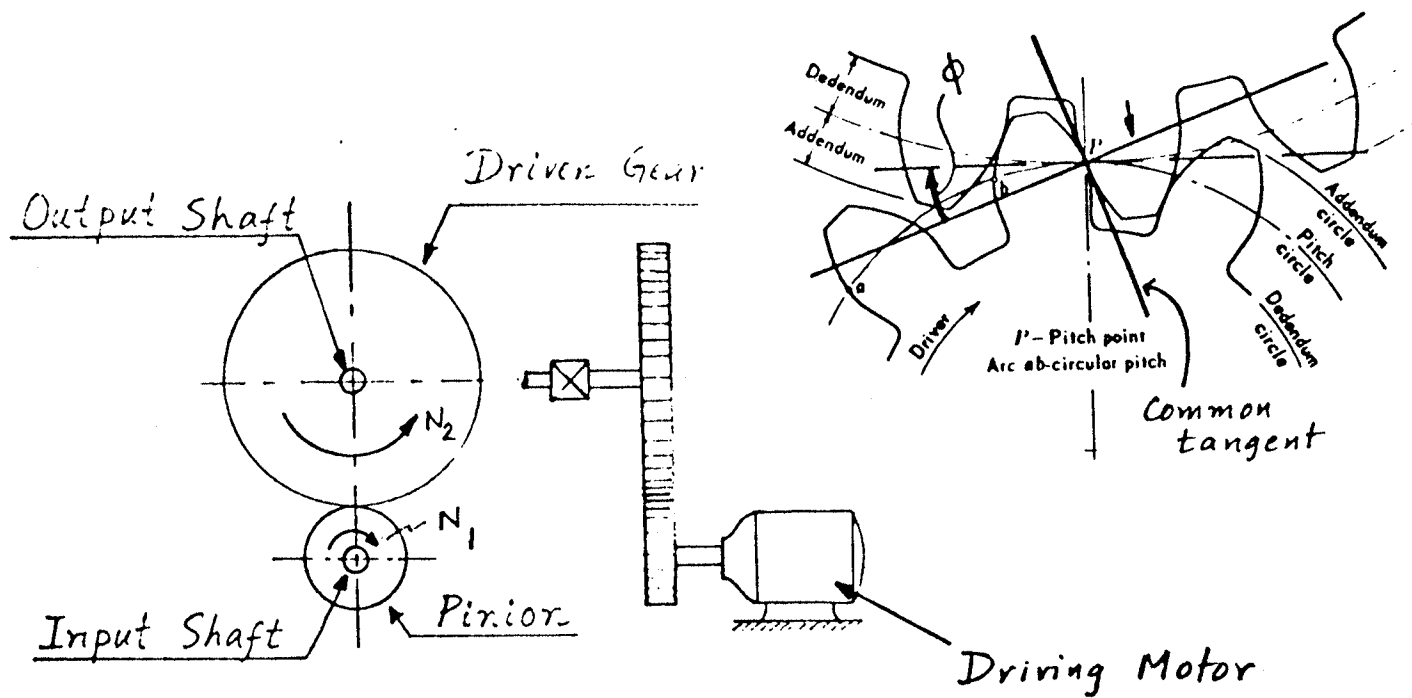


Fig. E 1.5

shaft and its key and the reactive tooth force exerted by the driven gear. The tooth force acts at an angle ϕ with the common tangent to the two pitch circles. This angle is called the pressure angle.

The forces acting on the driven gear are the tooth force exerted by the pinion and the reactive forces from the output shaft and key. By action and reaction, the tooth forces acting on the pinion and the driven gear are equal, opposite, and collinear.

Example E 1.6: Slider-crank mechanism of a single-cylinder air compressor.

This example illustrates how the pressure load from the compressed air acting on the piston results from the input torque to the crank shaft, using the principle of action and reaction.

1.7 FREE BODY DIAGRAM

Machines and structures are usually made of numerous components (members). A machine may be assembled using a housing and several shafts, pulleys, gears, linkages, as well as bolts, pins, keys, bearings, etc. A structure may be constructed using trusses, columns, beams, etc., connected by welds, rivets, bolts, etc. If we wish to determine the loads acting on or transmitted by a given component, we must conceptually isolate it from all components that are in direct contact with it. However, the action of the contacting components must be represented by the loads each transmit to the given component so that static equilibrium conditions are comprehensible. A sketch of a component conceptually isolated from contacting components for the purposes of equilibrium analysis is called a free body diagram.

Design analysis for a component starts by sketching its free body diagram. This diagram displays the isolated component along with a modeling of all external loads acting on it. External loads such as the weight of the component and applied service loads are generally regarded as known. Reactive loads transmitted to the component by physical contact with its neighboring components are usually unknown, and their magnitudes and directions must be determined by equilibrium analysis. It shall be demonstrated later that a correct free body diagram, including all relevant dimensions, is an absolute prerequisite for calculating the unknown reactive loads acting on any component of interest.

1.7.1 Examples (Friction Ignored)

The following examples are specifically chosen to simplify the modeling decisions required to draw the necessary free body diagrams. For example, we ignore friction in modeling the contact forces between members. We assume that the cross-sectional area of a cable is so small that we can model the static equivalent of its distributed internal force using a concentrated force vector which acts along the centerline of the cable. We assume that spheres (cylinders) make contact with other solids at points (along lines) and that the direction of the common tangent at the "point" of contact is obvious by inspection. The actual details of cable and spring attachments are suppressed, and so forth. Note the application of the principle of action and reaction in drawing free body diagrams.

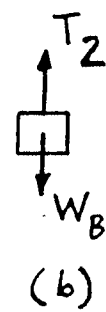
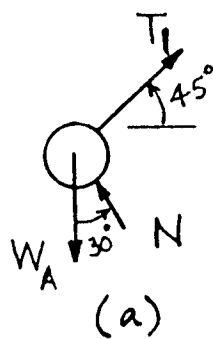
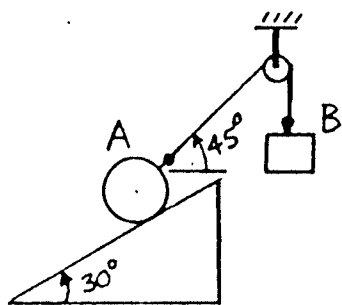


Fig. E 1.7

Example E 1.7: Free Body Diagram (Two-dimensional)

A homogeneous spherical ball A of weight W_A is connected by a flexible cable to a block B of weight W_B , and rests on an inclined plane, Figure E1.7. Draw free body diagrams for ball A and block B. Neglect friction.

Solution: The ball rests on an inclined plane and is connected to a cable. If we isolate the ball from the inclined plane and the cable, we will obtain its free body diagram, Fig. E1.7(a). Since the cable is a flexible member, i.e., cannot resist bending, it can transmit only a tensile force acting along its taut length. (A compressive force would cause the flexible cable to buckle.) The tensile force that the cable exerts on the ball is denoted T_1 . The reaction of the inclined plane on the ball is modeled using the normal force N . Note that this normal reaction acts perpendicular to the common tangent to both idealized contacting surfaces. Assuming the ball is homogeneous, the force of gravity (weight) W_A , is modeled as acting vertically downward through its geometric center, C.

In Fig. E 1.7(a), W_A is the only known external force. The directions of T_1 and N are known, but their magnitudes must be determined by static equilibrium analysis.

Block B is connected to the cable only. Therefore, if we isolate it from the cable, we will obtain its free body diagram, Fig. E 1.7(b). The cable exerts a vertically upward force T_2 on block B, whereas gravity acts vertically downward. The magnitude of T_2 is unknown. But if the pulley bearing is assumed to be frictionless, then by action and reaction (and intuition), T_2 in Fig. E 1.7(b) = T_1 in Fig. E 1.7(a).

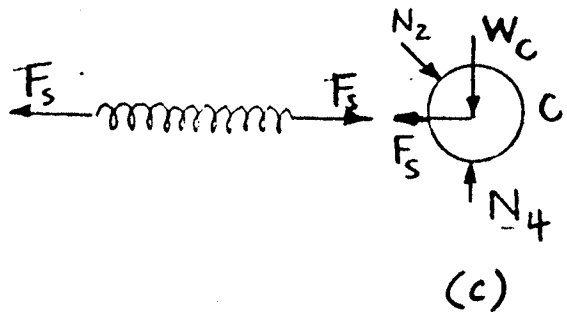
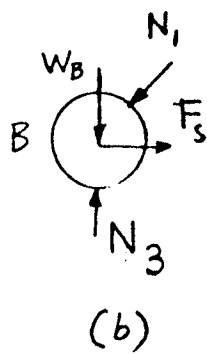
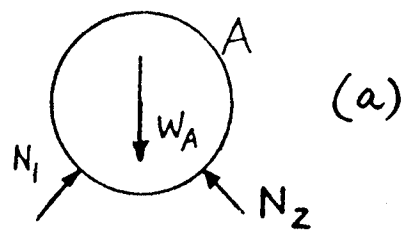
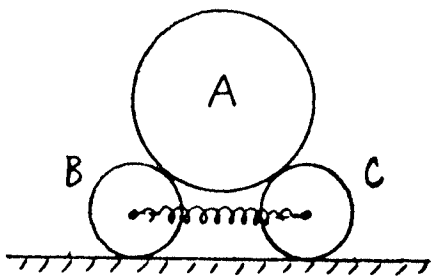
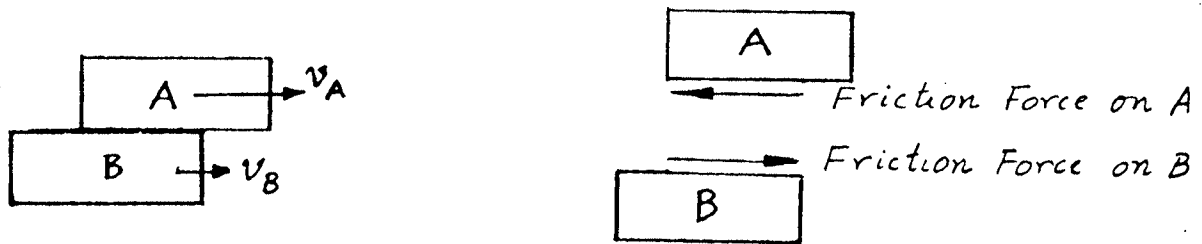


Fig. E 1.8

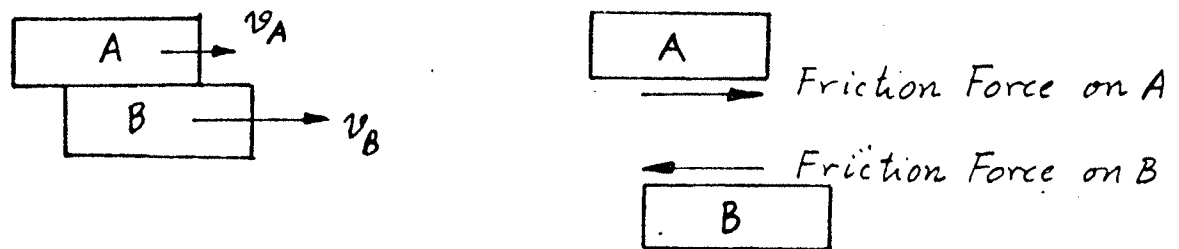
Example E 1.8: Free Body Diagram (Two-dimensional)

A large cylinder A rests on two smaller cylinders B and C which are connected to each other by coil springs in front and back. Draw free body diagrams for each cylinder and for the springs. Ignore friction.

Solution: The free body diagram for cylinder A, Fig. E1.8(a), includes the force of gravity (its weight is known) and the normal reactions from the two smaller cylinders, N_1 and N_2 . The free body diagram for cylinder B includes the force of gravity (its weight is also known), the normal reactions from the base plate and from the large cylinder, W_B , N_3 and N_1 , respectively, as well as the spring force. The coil springs keep the two smaller cylinders from moving (further) apart, and thus experience tensions which are modeled by tensile forces acting along their centerlines.



(a) $v_A > v_B$



(b) $v_A < v_B$

Note: Only Friction Forces are shown

Fig. 1.17

1.7.2 Examples (Friction Included)

If friction is considered in our analysis, we must indicate its direction (sense) correctly in each free body diagram. The direction of the friction force is such that it opposes sliding or impending sliding of one surface relative to the other. Consider, for example, blocks A and B in Fig. 1.17(a), which are in contact with each other and are moving in the same direction, but with different velocities. Suppose that the velocity of A is greater than the velocity of B. To an observer located on block B (who has the same velocity as B), block A would appear to move to the right. Since motion of block A relative to block B is to the right, the friction force exerted by block B on block A will be directed to the left. On the other hand, to an observer located on block A, block B would appear to move to the left. Since motion of block B relative to block A is to the left, the friction force exerted by block A on block B will be directed to the right.

Now, consider the case when the velocity of block A is less than the velocity of block B, Fig. 1.17(b). Again, by determining the sliding motion of block A relative to block B, we see that the direction of the friction force exerted by block B on block A is to the right. The opposite is true for block B.

If the blocks are not actually in motion, we determine the direction of the friction force by examining the tendency for sliding between the respective blocks.

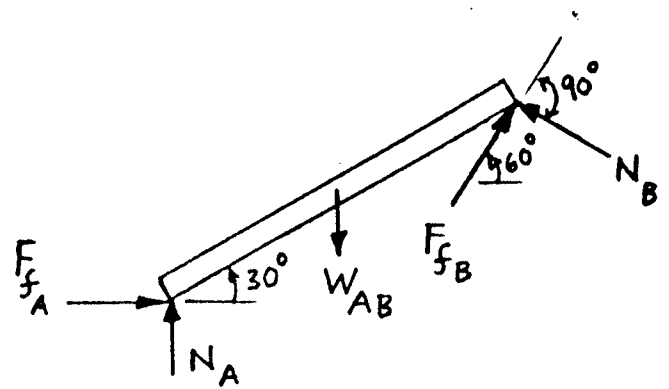
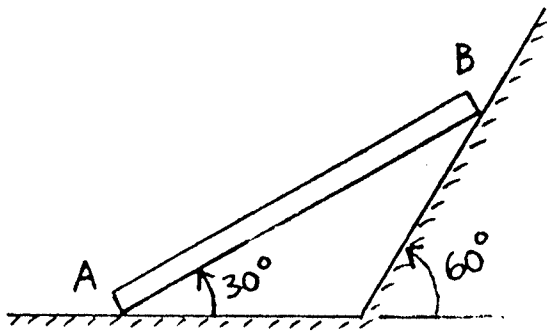
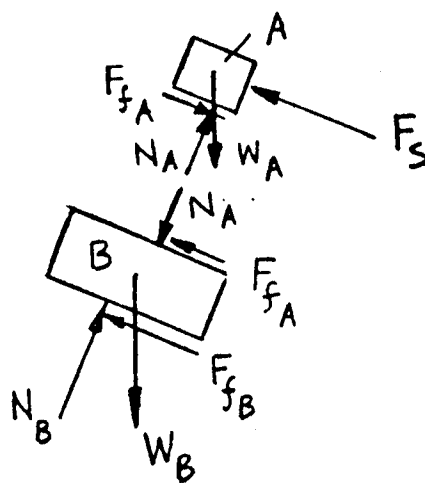
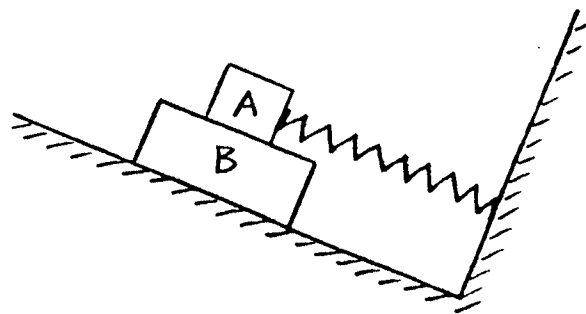


Fig. E1.9

Example E 1.9 Free Body Diagram (Two-Dimensional) with Friction

A homogeneous rod AB, of weight W rests on a horizontal floor at end A and against an inclined plane at end B. Draw the free body diagram for the rod. Include friction.

Solution: Fig. E1.9(a) presents the free body diagram for rod AB. Since end A of the rod has a tendency to slide to the left relative to the floor, the friction force F_{fA} of the floor acting on the rod is directed to the right. Simultaneously, the rod has the tendency to slide downward at end B relative to the stationary inclined plane; therefore, the friction force F_{fB} of the inclined plane acting on the rod is directed upward along the plane. The force of gravity (ie., the weight of the rod) and the normal reactions, N_A and N_B at A and B respectively must also be included in the free body diagram. Rod weight, W_{AB} , is the only known magnitude in this example. Gravity is modeled as acting vertically downward through the geometric center of the bar, point C, on the basis of the assumption that the bar is homogeneous.



(a)

Fig. E 1.10

Example E 1.10: Free Body Diagram (Two Dimensional) with Friction

Two blocks of weights W_A and W_B are placed on an inclined plane. Block A rests against a coil spring as shown. Draw free body diagrams for blocks A and B. Include friction.

Solution: Block B, in this example, is restrained from sliding down the inclined plane by frictional forces generated between its bottom surface and the surface of the inclined plane, as well as between its top surface and block A. Since block B has a tendency of sliding downward relative to both block A and the inclined plane, both these frictional forces are directed upward as shown in the free body diagram of block B, Fig. E 1.10(a). To an observer located on block B, block A would appear as moving upward. Therefore, the frictional force of block B acting on block A is directed downward. Note that the same conclusion can be reached by using the principle of action and reaction.

The free body diagram for block A also includes the force of gravity W_A , the normal reaction force N_A and the compressive spring force F_s . The free body diagram for block B includes the force of gravity W_B and the normal reaction forces, N_A and N_B .

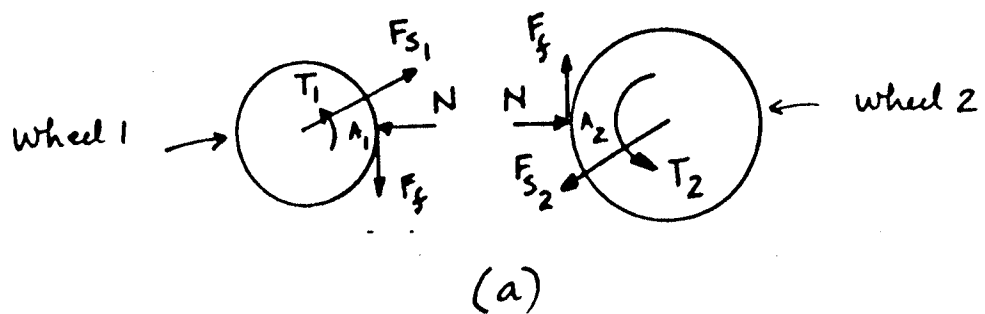
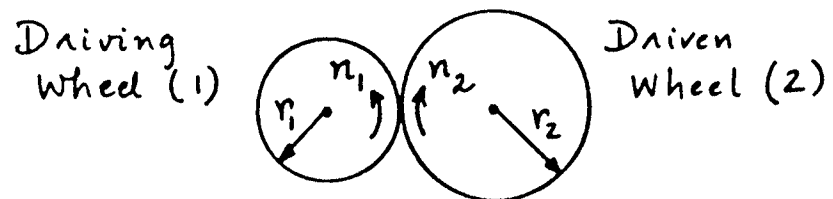


Fig. E 1.11

Example E1.11 Direction of Frictional Forces in a Friction Drive

One simple way of transmitting power from one shaft to another is by means of using a pair of friction wheels. Fig. E1.11 shows a friction drive in which wheel 1 is the driving wheel mounted on the input shaft and wheel 2 is the driven wheel mounted on the output shaft. The power that the friction drive transmits depends on the magnitude of the friction force that can be developed between the two wheels. Draw the free body diagram for each wheel.

Solution:

If there is no slip between the two wheels, the instantaneous velocities of contact points A_1 and A_2 must be identical. Therefore,

$$v_A = r_1 (2\pi n_1) = r_2 (2\pi n_2)$$

which gives

$$n_2 = \left(\frac{r_1}{r_2} \right) n_1$$

Suppose the frictional resistance between the wheels is not sufficiently large to prevent slipping, in which case $v_{A_2} < v_{A_1}$. To an observer located at point A_2 on wheel 2, point A_1 on wheel 1 would appear to move downward. Accordingly, the frictional force F_f of wheel 2 acting on wheel 1 acts downward, Fig. E1.11(a). Assuming that the weight of wheel 1 and its integral input shaft are negligible, the free body diagram for wheel 1 also includes the normal force, N , associated with the action of wheel 2 on wheel 1, the force F_B of its bearings acting on the input

shaft, and the input torque T_i .

The free body diagram for wheel 2 and its integral output shaft can now be established by action and reaction (for F_f and N) and by modeling the action of the bearings and the output coupling on the output shaft, Fig. E1.11(b). Observe that the output torque must oppose the rotation of the output shaft.

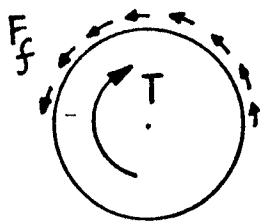
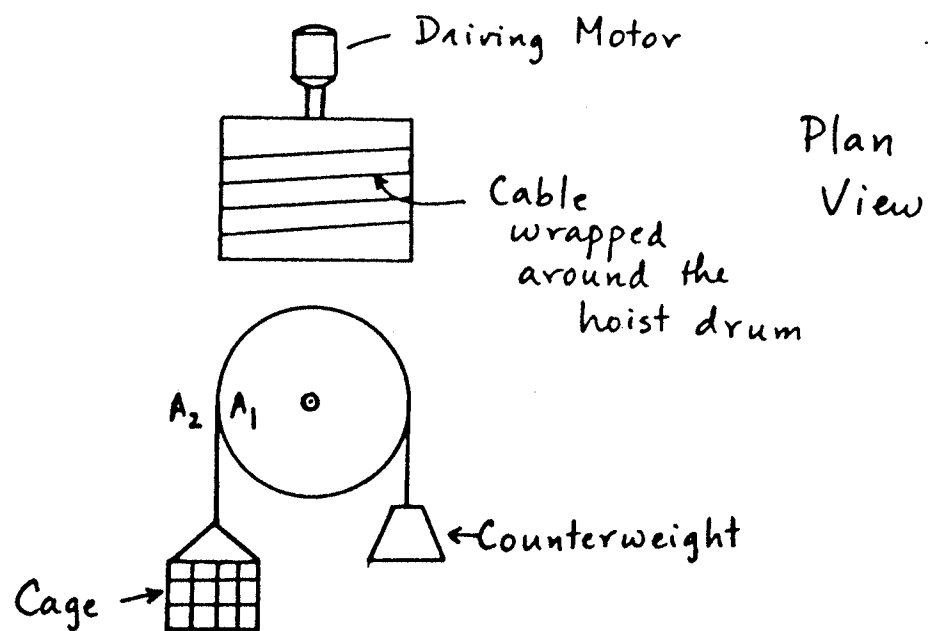
Example E1.12: Direction of Frictional Forces in a Traction Hoist

Consider the traction hoist in Fig E1.12, in which a hoist cage is suspended from a cable which wraps around a drum. A counterweight is suspended from the other end of the cable, so that the size of the driving motor and the braking system can be reduced. Determine the direction (sense) of the frictional force of the cable acting on the drum.

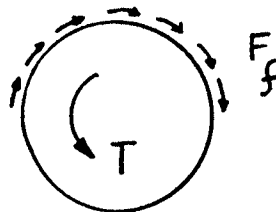
Solution:

Suppose the hoist cage is located at the bottom of its travel, just starting to move upward. Clearly, the drum must rotate in the clockwise direction. If there is no slip between the drum and the cable, the instantaneous velocity of all points of contact, e.g., A_1 and A_2 , must be the same. On the other hand, if the cable tends to slip relative to the drum, the drum would appear to rotate clockwise relative to the cable. Thus the frictional force of the cable on the drum acts counterclockwise, Fig. E1.12(a).

Suppose the drum brake is applied abruptly when the cage nears the top of its travel and that the cable again tends to slip, in which case it would appear to an observer located on the drum that the cable is rotating clockwise (relative to the drum). Thus, the frictional force of the cable on the drum acts clockwise, Fig. E1.12(b).



(a)



(b)

Fig. E1.12

PROBLEM SET ONE

P 1.1

For each of the following members and components, prepare appropriate sketches, describe and display the primary and secondary external loads that act during normal (i.e., reasonable) service operation. Consider, for example, loads due to gravity, internal and external pressure, thermal expansion, physical contact including friction, etc. Use your imagination, understanding and experience to formulate your answers. You may also survey literature, e.g., magazines, journals, handbooks, etc., available in any library.

- (a) a long segment of a buried natural gas pipe line
- (b) a front wheel of a rear-wheel drive automobile
- (c) a front wheel of a front-wheel drive automobile
- (d) a glass window in a tall building
- (e) a small electric hand drill
- (f) a submarine hull
- (g) a boiler in a power generating station
- (h) a common nail
- (i) an automotive thermostat
- (j) a short segment of the cable of an elevator
- (k) a tool bit in a lathe
- (l) the belt of a continuous conveyor
- (m) the frame of an arbor press
- (n) the crankshaft of an air compressor

The objective of this problem was to make you think about loads and to use judgment regarding what is and what is not important. We also hope that you observe that design solutions are open-ended in the context that

no analysis (not even the author's) is perfectly complete or completely accurate. Hopefully, however, our analyses will improve as we accumulate understanding and experience.

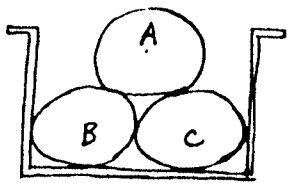
P 1.2

Prepare appropriate sketches and model, describe and illustrate the following actions (loads) according to the apparent area of contact (i.e., concentrated versus distributed loads). Describe the associated deformations of the underlined solid bodies in words and by appropriate sketches.

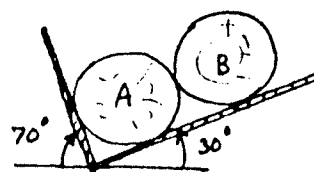
- (a) A 150 lbs. man walking on a 20 ft. long wooden plank to cross the creek. (A peg-legged man). (A dog).
- (b) Two tug-of-war teams pulling on a rope.
- (c) The action of cutting cheese with a knife.
- (d) The action of pulling a piece of paper through a set of fixed rollers. (What if the rollers are driving).
- (e) The action of tightening a screw using a screw driver.
- (f) The action of pulling a bent nail from a board using a claw hammer.
- (g) The action of cutting a piece of paper using a pair of scissors.
- (h) The action of opening a door by turning a door knob.
- (i) The action of opening a door by pulling on a door handle.
- (j) The action of inflating a balloon.
- (k) The action of books stacked on a book shelf.
- (l) The action of 40 mph wind on the vertical pole of a STOP sign.

- (m) The action of gripping a lawnmower handle.
- (n) The action of a rubber band tightly wrapped around a newspaper.

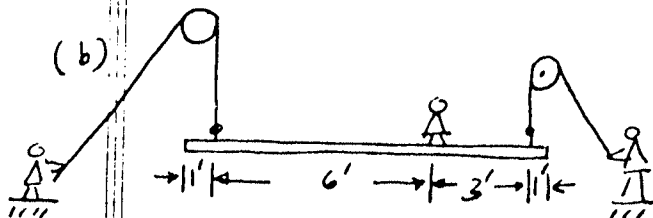
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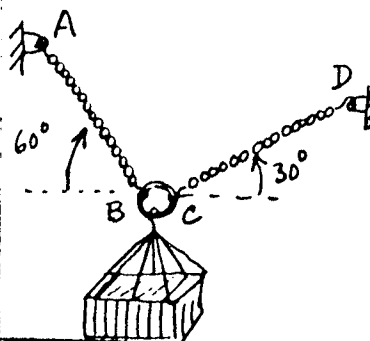
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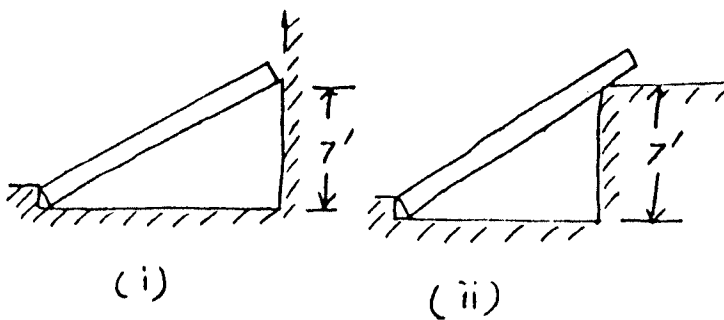
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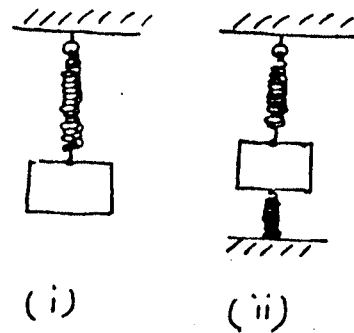
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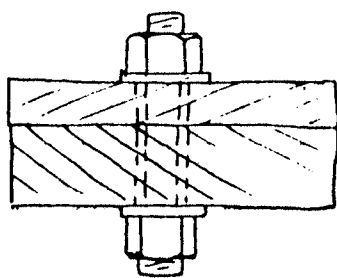
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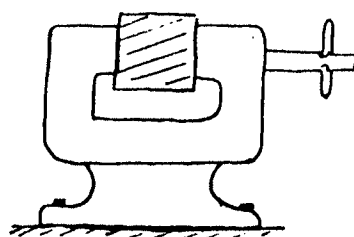
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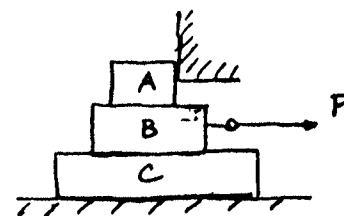
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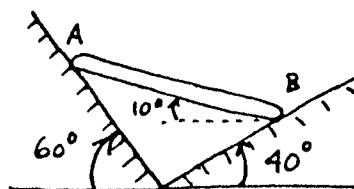
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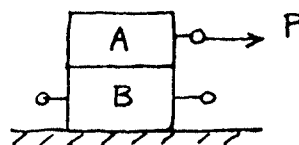
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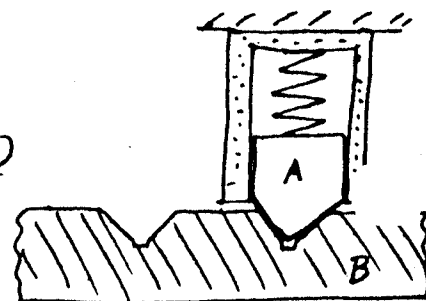
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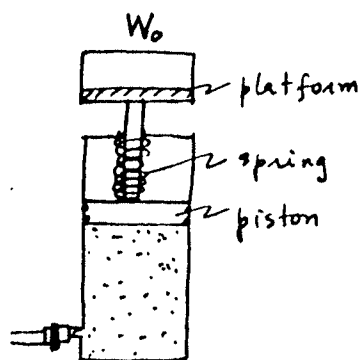
(k)



(m)



(h)



P 1.3

Draw the free body diagram (f.b.d.) for the member(s) indicated in each of the following problems. Neglect friction and member weight unless stated otherwise. Use the action and reaction principle where applicable.

- (a) Three identical cylinders, each weighing 4 lbs., placed longitudinally in a box. Draw the f.b.d. for each cylinder and for all three cylinders together.
- (b) A horizontal platform (weight = 50 lbs.), carrying a 200-lb. man, is being raised slowly. Draw the f.b.d. for the platform.
- (c) Two identical wooden logs, each weighing 100 lbs., are supported in a bin as shown. Draw the f.b.d. for each log.

- (d) A 250-lb. wooden crate is suspended from a steel ring by means of two cables AB and CD. Draw the f.b.d. for the ring and for each cable.
- (e)
- (i) (ii)
- A 15 ft. long wooden log, weighing 175 lbs., rests (i) against a concrete wall, (ii) on a concrete step. Draw the f.b.d. for the ladder in each case.
- (f) A metal block, weighing 1.5 lbs., is suspended in air by (i) a single spring, and, (ii) two identical springs as shown. Draw the f.b.d. for the block in each case.
- (g) Two metal blocks are clamped together by tightening the nuts at the ends of a threaded stud passing through holes in both blocks. Draw the f.b.d. for each block and the stud, with and without the nuts attached. Bonus: Include friction.

(h)

An air-cylinder is used to lift a large weight W placed on the platform. Draw the f.b.d. for the piston.

(i)

A metal block of weight W is squeezed between the jaws of a vice. Draw the f.b.d. for the block. (Hint: What keeps the block in place?)

(j)

A homogeneous rod AB (weight = 5 lbs.) is placed on two inclined surfaces as shown. Draw the f.b.d. for the rod, (i) neglecting friction, and, (ii) including friction.

(k)

Block A, weighing 10 lbs., is placed on block B, which also weighs 10 lbs. A force $P = 5$ lbs. is applied to block A. Draw the f.b.d. for both blocks in each of the following cases: (i) no force is acting on block B, (ii) a force $Q = 15$ lbs. is applied to block B in a direction opposite to that of P ,

(iii) a force $Q = 15$ lbs. is applied to B in the same direction as P.

(1)

Three boxes A, B, and C are arranged as shown. $W_A = 10$ lbs., $W_B = 15$ lbs., and $W_C = 40$ lbs. A force P is applied to box B. Draw the f.b.d. for each box. Include friction.

(m)

In a mechanical locking device, a wedge-shaped pin A is inserted in the mating slot of a component B. Draw the f.b.d. for pin A. Include friction.

P 1.4

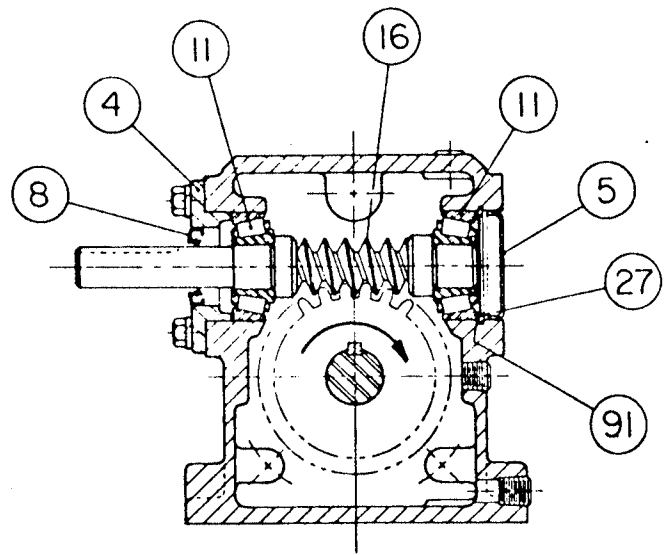
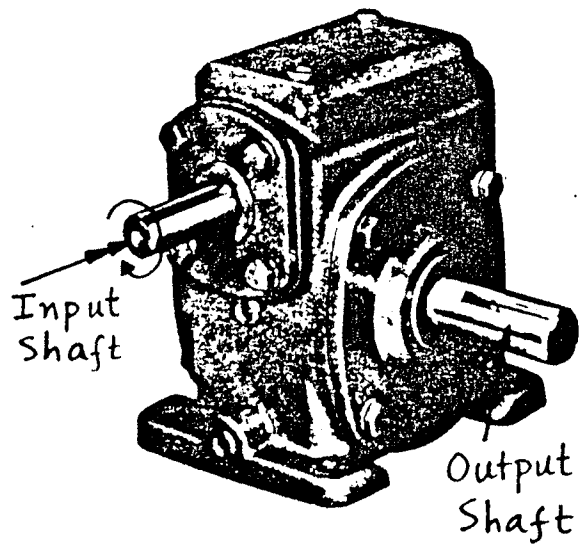
Indicate by appropriate sketches the directions (senses) of the input and output torques for the following power transmission mechanisms:

(a) Worm Gear Speed Reducer

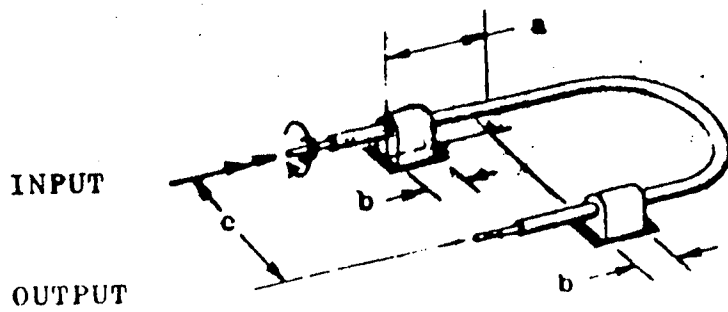
(b) Flexible Cable Arrangement

(c) Flexible Cable Arrangement

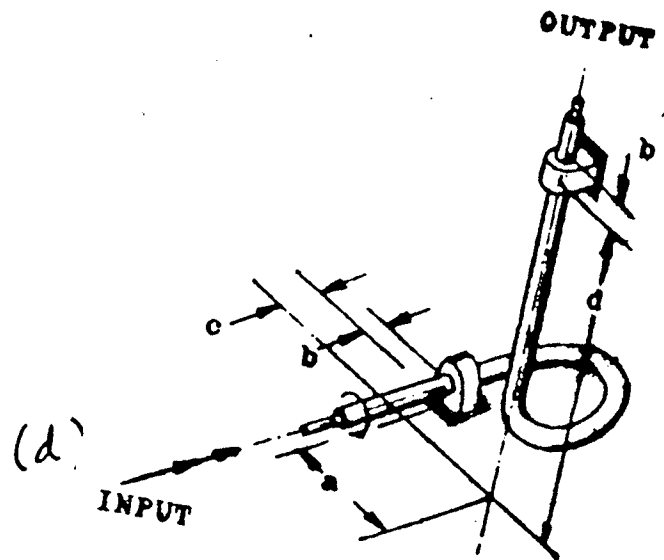
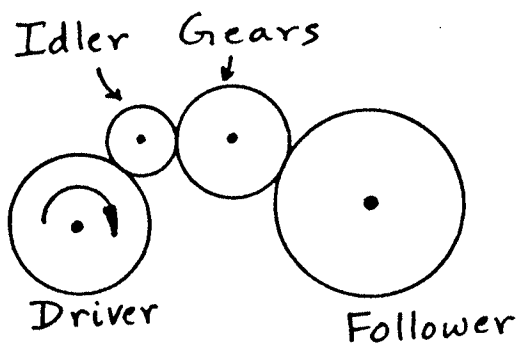
(d) Gear Train



(a)

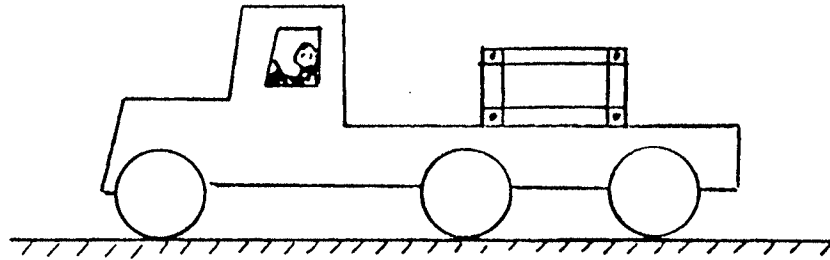


(b)



(d)

P 1.5



Consider a truck driving forward with a crate resting on its bed.

(a) Suppose the truck accelerates rapidly. What force causes the crate to accelerate also? If the crate slides, what is the direction of its motion relative to the truck bed. What is the direction of the frictional force of the truck bed on the crate? Of the crate on the truck bed?

(b) Suppose the truck decelerates rapidly and the crate slides forward. What is the direction of the frictional force of the crate on the truck bed? Of the truck bed on the crate?

(c) Suppose the truck makes a sharp turn. In which direction does the crate tend to slide? What is the direction of the frictional force of the truck bed on the crate? Of the crate on the truck bed?

(d) Suppose the truck is climbing a hill. Is the crate more likely or less likely to slide as it accelerates? As it decelerates? Or makes a turn?

(e) Suppose the truck is going down a hill. Is the crate more likely or less likely to slide as it accelerates? As it decelerates? As it makes a turn?

(f) Suppose the truck accelerates so fast that its rear wheels spin. At the "point" of contact between the tire and the ground, what is the motion of the tire relative to the ground? What is the direction of the frictional force of the ground on the tire?

(g) Suppose a truck brakes so rapidly that its rear wheels skid. At the "point" of contact between the tire and the ground, what is the motion of the tire relative to the ground. What is the direction of the frictional force of the ground on the tire. Can the tire skid while it is still rolling, or must the wheels be "locked" for skidding to occur? Explain.

(h) Suppose the truck was forced to stop in traffic while climbing an icy hill. When it starts to move forward again, its wheels spin and the truck (a) slowly climbs the hill, (b) slowly slides backwards down the hill despite the "forward" rotation of the wheels. What is the direction of the friction force of the icy ground on the tires in (a)? In (b)? Explain the difference between (a) and (b) if any.

(i) Suppose the truck is just driving along a level road at a constant velocity. What is the direction of the frictional force of the ground on the tires. Explain.

PROBLEM SET TWO

Design modeling and analysis requires a critical perspective to discern subtle differences between the actual and idealized situations. Competent evaluations are enhanced by adopting a "devil's advocate" or "what if" tack. The purpose of this problem set is document that our Chapter One examples and explanations, however well intentioned, are naive and flawed.

P 1.6

Reconsider the plate bending fatigue test example, Example E1.1.

- (a) How is the height of the grip used to clamp end A adjusted so that it coincides exactly with the center of the stroke of the connecting rod. (Otherwise, the cyclic load has a mean component which is unwanted and unknown.)
- (b) No specimen is ever perfectly straight or flat. How are departures from flatness (anticlastic curvature and camber) taken into account and measured?
- (c) Does the speed of loading affect the deflection? Consider the thickness of the oil film in a journal bearing as well as "inertial" effects of moving parts. (Deflection measurements are made statically by turning the crank or cam to the position of interest by hand....because it is very difficult to do otherwise.)
- (d) Does the way the machine is mounted affect the deflection? (Suppose the machine is mounted on a very heavy table versus a small table. Suppose the mounting somehow involved "vibration

isolation" mounts or pads.)

(e) Bonus: You ask the question and then answer it.

P 2.2

Reconsider the ball bearing example, Example 1.2.

(a) Does the size and shape of the shaft and the bearing location affect the bearing load modeling? (Hint: Does the slope of the shaft due to its deflection affect the location of the contact between the balls and the inner and outer races?)

(b) Is the bore of the inner race of the bearing perfectly round and perfectly concentric with the inner raceway? Is the mating diameter of the shaft perfectly round? Do either have any burrs or nicks or other surface imperfections?

(c) When the inner race is pressed on the shaft, is the technique or procedure important? Could the bearing be started crooked?

(d) Are all of the ball exactly the same size? How would this affect Fig. E2.1(c)? [Bonus: Look up "signature analysis" as related to speed reduction gears on submarines and state the analogy, however theoretical.]

(e) Bonus: Make up your own question and then answer it.

P 2.3

Reconsider your own solutions to Exercise Set One Problems

(a) P1.1(a) through (n)

