**Incremental Parameter Feedback Formalism**

**Objective**
For design purposes a simplified approximate analysis of an electronic circuit often is a productive preliminary procedure. However even an approximate procedure must at least rest on a reasonable theoretical foundation. This note presents the mathematical formalism for the feedback concept for linear systems (including incremental parameter circumstances). The intention is less a prescription for analysis of circuits than the use of the formalism to provide insight into the application of feedback in electronic circuits. The constraint to linear circuits includes application to incremental parameter analysis.

**Introduction**
The block diagram circuit configuration drawn to the right is used to introduce the discussion. The voltage source and impedance on the left represent the Thevenin equivalent of an input source circuit. The impedance to the right is the 'load' across which the output voltage \( V_0 \) is measured. A two-port network is connected between the two to modify the signal transfer from the source to the load from what it would be with a direct connection.

The two-port network is presumed to be linear for this discussion, i.e., the input and the output voltage and current are related linearly. However linear or not there are two general constraints on these four terminal variables, corresponding to application respectively of KVL and KCL (alternatively conservation of energy and charge). For our purpose we need not specify these constraints in detail; it is sufficient here simply to recognize that because of them only two of the four terminal variables are independent. Which of the two variables are independent (so making the remaining two variables dependent) is a matter of how one chooses to describe the relationship between the variables; the relationship between the variables is itself fixed by the circuit. Whatever description is advantageous in a particular context may (should!) be used. Indeed part of the purpose of this discussion is to provide general criteria on the advantages and benefits of the different choices, which then may be used to guide a selection.

**Admittance Parameters**
Initially, and as will be seen in the course of the discussion without really introducing any basic limitations, we designate the terminal voltages \( e_1 \) and \( e_2 \) as the independent variables. Given the assumed linearity and passivity of the network, the relationship between the dependent terminal voltages and the independent terminal currents may be written mathematically as shown below, and also presented schematically as an equivalent circuit:

\[
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} =
\begin{bmatrix}
y_{i} & y_{r} \\
y_{f} & y_{o}
\end{bmatrix} \begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

\[
i_1 = y_{i} e_1 + y_{r} e_2
\]
\[
i_2 = y_{f} e_1 + y_{o} e_2
\]
\[
v_L = -\frac{i_2}{e_2}
\]

The subscripts for the coefficients are chosen for convenience. Thus ‘i’ refers to ‘input, i.e., it denotes the coefficient relating the dependent and independent input variables. Similarly ‘o’ relates the dependent and independent output variables. The subscript ‘f’ denotes the ‘forward’ transfer relating the dependent output variable to the input independent variable. And ‘r’ denotes the ‘reverse’ transfer from output to input. The coefficients of this particular description of the network are known generally as ‘admittance’.
parameters, or collectively as the admittance parameter matrix. The input and output coefficients actually have the units of admittances. The ‘short-circuit’ input admittance, i.e., the admittance measured at the input with $e_2$ set to zero, is $y_i$. Similarly the ‘short-circuit’ output admittance, i.e., the admittance looking into the output with $e_1$ set to zero, is $y_o$. The other two parameters are 'transadmittances', i.e., 'transfer' admittances; they relate a current at one pair of terminals to the voltage at the other pair. Both coefficients may be measured by appropriate short-circuit prescriptions similar to those described for $y_i$ and $y_o$.

One further constraint is now applied to the network; the voltage and current at the output are related by the constitutive relation for the load. For reasons to appear it is generally advantageous to describe the load as a ratio of output independent variable to output dependent variable or, more simply put, to have the same units as $y_o$; in this case $i_2 = -Y_L e_2$. Substitute this condition into the descriptive equations and calculate the transimpedance gain $G^{(Z)} = e_2/i_1$. There is a particular reason for this choice, to emerge in due course. For reasons to be made clear as part of the discussion make the calculation in two steps

First assume that $y_r = 0$, and calculate the ratio $G^{(Z)}_{NFB}$ (transimpedance without feedback), under the assumed condition. Recalculate for $y_r \neq 0$, designating the transimpedance in this case as the gain with feedback $G^{(Z)}_{FB}$. The purpose of this two step calculation is to express $G^{(Z)}_{FB}$ in terms of $y_r$ and $G^{(Z)}_{NFB}$. The equations show a superscript $(Z)$ to indicate that the ratio is that of the output voltage to input current. Other transfer ratios are considered later, distinguished by different superscripts.

Note: Although the equations do not seem to do so directly the source immittance actually may be included very easily. In this illustration consider the source as a Norton equivalent circuit with source admittance $Y_S$. This admittance is in parallel with $y_i$, and its contribution to the equations can be included simply by substituting $y_i + Y_S$ in place of $y_i$.

An additional relationship of interest shown above is the input admittance, also expressed via a two step calculation in terms of the admittance absent the feedback. A similar output admittance expression is considered later as an exercise.

The reverse transfer coefficient $y_r$ plays the role of the feedback sampling fraction used in earlier descriptions of feedback, i.e., it is a measure of the signal transfer from the output, here the output voltage, to the input, here the input current. Similarly $y_f$ describes the forward transfer of the network if there is no reverse transfer, i.e., no feedback.

Observe that, analogous to an earlier simplified discussion of feedback, if the ‘loop gain’ $y_r G^{(Z)}_{NFB}$ is much greater than 1 the gain with feedback approaches the inverse of the sampling fraction, i.e., $1/y_r$.

The admittance parameter description of the network is a mathematically precise formulation of the more descriptive earlier simplified presentation of the feedback concept.

**Illustrative Admittance Parameter Calculation**

An uncomplicated network serves for an illustrative calculation of the admittance parameters. The coefficients of the descriptive equations are constants (for a particular network) and so have a specific value no matter how calculated. In the case of admittance parameters perhaps the simplest calculation is the method of 'short-circuits'. Thus $y_i$ is the input admittance for $e_2 = 0$; hence $y_i = 1/R$. Similarly $y_o$ is the output admittance for $e_1 = 0$: $y_o = 1/R$. To calculate $y_f$ set $e_2 = 0$ and calculate the short-circuit output current; $y_f = -1/R$; note the negative sign corresponding to the voltage and current polarities defined. Similarly it may be determined that $y_r = -1/R$. 

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The equivalent circuit appears to be, indeed it is, more involved in this simple case than the circuit itself. However even so the equivalent circuit explicitly distinguishes the four circuit effects of the network. For example the voltage drop across the input involves both the direct effect of the input current (loading) and the effect of a reaction (feedback) from the output. The output voltage drop is associated both with an intrinsic loading effect at the output and a forward transfer from the input.

Incidentally this simple feedback circuit has been encountered before, and will be again later.
More Formalism
Suppose there are two networks each described by a set of admittance parameters, suppose the networks are connected so that they have common dependent variables, i.e., both have the same terminal voltages. The shunt-shunt input-output connection involved is sketched on the right. The admittance parameter descriptions (distinguished by superscripts a and b respectively) for each network are:

\[
\begin{bmatrix}
i_1^a \\
i_2^a
\end{bmatrix} =
\begin{bmatrix}
y_{i1}^a & y_r^a \\
y_f^a & y_o^a
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\quad \& \quad
\begin{bmatrix}
i_1^b \\
i_2^b
\end{bmatrix} =
\begin{bmatrix}
y_{i1}^b & y_r^b \\
y_f^b & y_o^b
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

The total input/output current is the sum of those for the individual networks, and the corresponding mathematical expression for this is:

\[
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} =
\begin{bmatrix}
i_1^a + i_1^b \\
i_2^a + i_2^b
\end{bmatrix} =
\begin{bmatrix}
y_{i1}^a + y_{i1}^b & y_{r}^a + y_{r}^b \\
y_{f}^a + y_{f}^b & y_{o}^a + y_{o}^b
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

A specific point to note is that the admittance parameters for the combined networks are, respectively, the sum of the parameters for the individual networks. It is important to recognize that this is not an accidental relationship but rather comes about because of the manner in which the networks are described, particularly the use of common independent variables. Keep in mind that the network properties here are the same as would be determined from any description; this particular description is designed to make certain relationships stand out.

The primary significance of this partitioning lies with consideration of the feedback term. A practical feedback design often involves a network ‘a’ for which in general \( y_{r}^a \ll y_{r}^b \), and so absent network ‘b’ network ‘a’ may be regarded as being ‘without feedback’. Network ‘a’ would be designed to provide high forward gain, and network ‘b’ provides the significant feedback. Then if \( y_{r}^b G_{NFB}^a >> 1 \) the combined networks present very closely the overall gain \( G^{(a+b)}_{FB} \approx 1/y_{r}^b \). Each network part can be designed separately for its specialized purpose.

Illustration
As an illustration we use a familiar circuit, whose performance is reinterpreted in the light of the preceding discussion. The inverting feedback amplifier previously considered may be thought of as a combination of two networks, a high-gain amplifier (possibly an opamp) and the shunt feedback resistor network. The admittance parameters of the resistor network may be calculated by the short-circuit method described before to determine \( y_i = y_o = 1/R \), and \( y_f = y_r = -1/R \).

When the networks are combined in a shunt-shunt connection the resistor network affects the amplifier in four ways, corresponding to the four components of the equivalent circuit. There is input ‘loading’, i.e., an admittance \( 1/R \) is added across the amplifier input. Similarly ‘output’ loading adds an admittance \( 1/R \) across the output, increasing the admittance seen looking back into the network. The forward transfer \( y_f \) of the amplifier is decreased by the term \( -1/R \). And finally the reverse transfer is decreased by the term \( -1/R \).

In a feedback design, particularly where the feedback network is passive, the feedback effect on the total forward transfer may be expected to be negligible. After all, to achieve meaningful feedback we must design the amplifier forward transfer to be large so that \( y_f G(Z)_{NFB} \gg 1 \). A different
expectation would be held for the reverse transfer, where the feedback network \( y_r \) parameter ordinarily would be designed to dominate that of the amplifier. These are precisely the assumptions used for the idealized opamp amplifier.

Input and output loading may or may not be important, depending on a particular circuit context. Fortunately however accounting for the loading is a relatively simple matter of adding the appropriate shunt \( y_i \) and \( y_o \) of the feedback network to the corresponding terms of the amplifier.

If the feedback condition is met, i.e., \( y_r G_{NFB} \gg 1 \), then \( G_{FB} = 1/y_r = -R \) as expected from an idealized opamp feedback analysis. Note that the input admittance is increased (or input impedance decreased) to the same extent that \( y_r G_{NFB} \gg 1 \). The feedback actually improves the high-gain amplifier approximation for which the input is considered a virtual short-circuit.

**Generalization**

Before actually generalizing the formalism it is helpful to introduce some terminology which emphasizes relationships. Since the feedback is designed to make the overall network gain \( G_{FB} = 1/y_r \) it follows that the natural gain parameter to use corresponds to this relationship. For the admittance parameters this is the ratio of the output voltage to the input current. However emphasis should be placed on the relationship rather than simply on memorizing the appropriate gain variable. This gain is natural in the sense that one expects to vary the input independent variable to modify the output dependent variable, and then sample the dependent output variable to use to modify the independent input variable.

In general, using similar reasoning, the natural input immittance is the ratio of the input independent variable to the input dependent variable, and the natural output immittance is defined similarly.

The reason for introducing the terminology is to provide a means to refer to various feedback arrangements without actually having to specify the precise arrangement to be used. This is illustrated in the next section.

**‘Open-Circuit’ Impedance Parameters**

We generalize the preceding discussion by example; suppose we now choose as independent variables the terminal currents rather than the voltages as before. The descriptive equations then take the form:

\[
\begin{align*}
\begin{bmatrix}
  e_1 \\
  e_2 \\
\end{bmatrix} &=
\begin{bmatrix}
  z_i & z_r \\
  z_f & z_o \\
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
\end{bmatrix} \\

i_1 &= z_i \cdot i_1 + z_r \cdot i_2 \\
\begin{bmatrix}
  e_1 \\
  e_2 \\
\end{bmatrix} &=
\begin{bmatrix}
  z_i & z_r \\
  z_f & z_o \\
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
\end{bmatrix} \\
Z_L &= -\frac{i_2}{e_2}
\end{align*}
\]

A point to be stressed here is the formal similarity between these equations and the admittance parameter description previously considered. The purely symbolic algebraic manipulations used to obtain previous equations would when applied to the present situation result in similar equations. The difference lies only with the symbols used, and that can be taken into account simply by substitution of corresponding symbols.

Thus, for example, the ‘natural’ gain would be the transadmittance, i.e., the ratio of the output current to the input voltage. On substituting symbols if the admittance parameter expressions the impedance

\[
\begin{align*}
\frac{Z_N_{NFB}}{Z_{N_{NFB}}} &= \frac{-z_f}{z_i (z_o + Z_L)} \\
\frac{Z_N_{FB}}{Z_{N_{FB}}} &= \frac{\frac{z_f}{Z_N_{NFB}}}{1 + z_r \cdot \frac{z_f}{Z_N_{NFB}}}
\end{align*}
\]

\[
Z_{IN\{FB\}} = z_i (1 + z_r \cdot \frac{Z_N\{FB\}}{z_o})
\]
The overall network is imagined to be formed from two parts, the natural connection of the parts, the connection that provides both parts with the same independent variables, is a series-series connection as shown to the left. The ‘z’ parameters for the resistor feedback network shown are \( z_i = z_o = z_f = z_r = R \). Assuming the loop gain \( z_r G_{Y_{NFB}} \gg 1 \), and that \( z_f \) of the feedback network dominates that of the amplifier, \( G_{Y_{FB}} = 1/R \). A more physical derivation, assuming the amplifier is an opamp, recognizes that the voltage drop across R is essentially the input voltage, and the output current is forced to be \( 1/R \) times this voltage. Incidentally note the polarity for the amplifier input terminals.

The input loading merely increases the input impedance somewhat over that of the opamp alone. The output loading may be more significant, depending on the context. For 'negative' feedback \( z_r G_{Y_{NFB}} > 0 \), and since \( z_r > 0 \) so also should be the gain.

**Hybrid Parameters**

A convenient choice of independent variables for many purposes is the input current and the output voltage; the letter ‘h’ commonly designates these hybrid parameters. Note the mixed dimensions among the hybrid parameters.

Based on the previous discussion the natural gain, i.e., the gain that the feedback will stabilize, is the voltage gain. The hybrid feedback samples the output voltage and feeds this back to modify the input voltage. Assuming a partitioning between two networks as before the appropriate connection is a series connection on the input and a shunt connection on the output, as shown below, left; the corresponding descriptive equations are shown to the right.

The hybrid parameters for the illustrative feedback network are \( h_i = R_1 || R_2 \), \( h_o = R_1 + R_2 \), \( h_r = h_f = R_1/(R_1 + R_2) \). If the amplifier voltage gain is high enough \( G(V_{FB}) \to (R_1 + R_2)/R_1 \). The feedback network will load the amplifier with a series resistance \( R_1 || R_2 \) on the input, and a shunt resistance \( R_1 + R_2 \) on the output.

An illustrative calculation for this feedback configuration is presented as a problem.

**Inverse Hybrid Parameters**

Of occasional use is the choice of independent variables converse to the hybrid pair. The independent variables are the input voltage and the output current. The natural gain is current gain, and the descriptive equations are as drawn to the left.
INSTRUCTIONAL EXERCISES

These exercises are intended more as illustrations than design or analysis exercises, i.e., as an extension of the preceding discussion. Where transistors are involved use 2N3904, 2N3906.

1) The circuit for a two-stage incremental signal amplifier is drawn to the right. Estimate the DC bias voltages and currents using methods discussed elsewhere (neglect the Q2 base current compared to the Q1 collector current, and write a KVL expression through the Q1 collector resistor, the Q2 emitter junction, the Q1 base resistor, . . .). Use PSpice to verify the estimate.

From the DC calculations obtain the incremental parameter equivalent circuit as shown below. The amplifier may be interpreted as having series-shunt feedback applied as shown enclosed by the dotted lines on the circuit diagram. Since the independent variables should be the input current and the output voltage the 'natural' gain for this feedback connection is $G(V)$.

An estimate of the amplifier gain is available by viewing the circuit as indicated to the left and assuming (roughly) idealized opamp behavior. The estimated $G(V)_{FB}$ is $1+(10/0.1)=101$ (40 db). A PSpice computation using 2N3904 transistors computed 39.25 db. A more detailed incremental parameter analysis may be made as follows.

The incremental equivalent circuit is drawn below; the base-emitter resistances are to be calculated from the transistor bias currents. First calculate $G(V)_{NFB}$, the gain without feedback, i.e., with $h_r = 0$. Although it may be useful in an appropriate context to do so it is not formally correct simply to remove the feedback resistors from the circuit and assume this corresponds to setting $h_r$ to zero. The feedback term $h_r$ is indeed set to zero but removing the resistor effectively sets all the hybrid parameters to zero. This approximation has been used before in amplifier design notes, but always with a qualification assuming the amplifier to which the feedback is applied is closely enough an 'idealized' amplifier. In circumstances where the feedback is significant this is usually a reasonable approximation; even with respect to the feedback input and output loading performance considerations often subordinate these factors.

The hybrid parameters for this feedback network were calculated before. It is not necessary actually to compute the corresponding parameters for the amplifier proper. The loading contributions of the feedback network simply may be added to the circuit topographically. Thus, for example, the input loading corresponds to inserting in series the parallel combination of the feedback resistors ($h_i$), and the output loading ($h_o$) corresponds to the a shunt connection of the series combination of the two resistors. The loading is shown in the incremental circuit diagram below. The contribution of $h_f$ may be safely neglected; the forward transfer of the passive feedback network is certain to be considerably smaller than the amplifier forward transfer if the feedback is significant (and if it is not the point of the formalism disappears).

It is clear that only the feedback network provides a reverse transfer; there is no reverse signal path in the amplifier proper. (This is not always true; for example the base-collector capacitance of the BJT could be significant at high enough frequencies.) Analysis of the circuit to calculate $G(V)_{NFB}$ is straightforward; the
calculated value is 1442. Then \( h_r G(V)_{NFB} \approx 14.3 \). Hence \( G(V)_{FB} = 1442/(1+14.3) = 94.2 \). A PSPICE analysis computes a gain of 89.3. Compare this to the 'quick' estimate of 101.

The circuit is uncomplicated enough so that a straightforward nodal analysis (say) is quite feasible. Alternatively transform the 100\( || \)10K resistor into the base and use a 'ladder' calculation. However the detailed calculation is less important numerically than the illustration of the accounting for the loading effect of the feedback network. Where a amplifier design is involved the approximate analysis is useful for providing a prototype circuit, and for suggesting the relative importance of different elements in affecting the circuit performance.

The calculation of the DC bias currents and voltages involves the portion of the circuit illustrated to the right. As described elsewhere the Q2 base current is neglected (to be verified) and a loop equation for the Q1 collector current \( IC_1 \) is

\[
9 = IC_1(3.3) + 0.7 + (IC_1/120)(47) + 0.7 + (121/120)IC_1(0.1).
\]

The transistors are assumed to be 2N3904 with a nominal \( \beta \) of 120. Note that the biasing limits the influence of \( \beta \), and a precise value is not necessary. Calculate \( IC_1 = 2 \text{mA} \). Use this value to estimate the Q2 emitter voltage, and so the emitter current of Q2; \( IE_2 = 1.7 \text{mA} \). PSpice computes values of 2.04\text{mA} and 1.55\text{mA} respectively. The currents may be used to estimate \( r_{be1} \approx 1.5K \Omega \), and \( r_{be2} \approx 1.8K \Omega \). As noted the actual calculations using the incremental circuit are straightforward; the essential point is the appreciation of the feedback aspects.

2) The circuit for a DC-coupled two-stage incremental signal amplifier is drawn below. The DC current through the 2.7 K\( \Omega \) feedback resistor changes in proportion to the Q1 collector voltage, since the junction voltages will vary little. This current is used to modify the Q1 base current, decreasing the base current if the collector current increases to provide degenerative feedback. Anticipate (and verify subsequently) that the Q2 base current may be neglected compared to the Q1 collector current. Then write a KVL equation (in terms of the Q1 collector current) through the collector resistor, the Q2 emitter junction, the 2.7K\( \Omega \) feedback resistor, … . Note that the current through the 2.7K\( \Omega \) consists of the Q1 base current plus the DC current across the 1 K\( \Omega \) source resistance associated with the Q1 junction voltage. Estimate the DC bias voltages and currents using methods discussed elsewhere (neglect the Q2 base current compared to the Q1 collector current, and write a KVL expression through the Q1 collector resistor, the Q2 emitter junction, the Q1 base resistor, etc.). Use PSpice to verify the estimate. Although the Q2 collector is indicated as the output terminal we may take the emitter as the output for purposes of calculation; because the emitter and collector currents will be very nearly the same the emitter
The base-emitter resistances are calculated from the DC emitter currents. The DC collector current IC1 of Q1, assuming the base current of Q2 may be neglected by comparison, is obtained from

\[ 9 = 5.6 \times IC1 + 0.7 + \left\{ (0.7/1) + (IC1/120) \right\}(2.7) + 0.7 \]

and IC1 \( \approx 1.02 \) ma. Note that the current in the feedback resistor is not simply IC1/120. Use the calculated current to estimate the Q2 emitter voltage, and then the Q2 emitter current IE2 \( \approx 1.88 \) ma. PSpice values computed are 1.03 ma and 1.79 ma respectively. Then calculate rbe1 \( \approx 3.06 \) K and rbe2 \( \approx 1.66 \) K.

The natural gain for the shunt-shunt feedback connection is \( G_{FB}(Z) \), and the gain without feedback is calculated to be \( G_{NFB}(Z) = -300.16 \). Note: The voltage at the Q2 emitter is calculated to be -123.3; the input current is calculated as 0.41 ma. Then \( G_{FB}(Z) = -300.16/(1 + 300.16/2.7) = -2.68 \) KΩ. A PSpice computation gives \(-2.64 \) KΩ for the gain (to the Q2 emitter).

The Q2 incremental emitter current is essentially equal to the collector current, and so the incremental voltage at the collector is to the incremental voltage at the emitter in proportion to the resistance ratio involved. The idealized amplifier approximation assumes the base current for Q1 is small compared to the input current. Since the gain without feedback is much higher than the gain with feedback this will be a good approximation in the present case, i.e., assume the input current flows almost entirely through the 2.7 KΩ feedback resistor. Write a node equation at the Q2 emitter using this assumption: \( 1 + (121)ib + (2.64/2.2) = 0 \) or \( 120ib = -2.18 \), and so transimpedance gain to the Q2 collector is \(-2.18)(2.2) = -4.8 \). PSpice provides a value of \(-4.76 \).

3) In addition to ‘local’ series-series feedback in the individual stages the amplifier circuit drawn to the right involves an overall series-shunt feedback. By inspection roughly estimate the voltage gain of the amplifier absent the series-shunt feedback (but with the loading considered) to be \( \approx 800 \). (Describe the basis for how you make the estimate.) Estimate (roughly) by inspection the overall voltage gain with feedback to be 11. (Describe the basis for how you make the estimate.) Use an incremental parameter analysis to make more precise gain estimates. Finally, compare estimates with a PSpice analysis.
An estimate of the AC voltage gain as $1 + (1/1) = 11$ can be obtained by assuming the noninverting amplifier around which the feedback is applied has idealized amplifier behavior.

Use a simplified PWL transistor model to estimate the DC bias currents and voltages. An exact analysis of the PWL circuit is not difficult but it is tedious. Instead advantage will be taken of the feedback property of reducing the effect of a perturbation; an iterated calculation can be used to converge from a guess onto a solution. Of course an educated guess, although not necessary, is helpful.

The Q1 emitter voltage is $(10)(10/110) - 0.7 = 0.21v$. If the feedback is effective (high gain amplifier) the ‘error’ current from the emitter would be zero; thus neglect the Q1 emitter current and assume the entire 2.1ma through the 100Ω emitter resistor is supplied through the 1KΩ feedback resistor. From this determine the Q2 collector voltage as $0.21 + 2.1 = 2.31v$, and so the Q2 collector current is $\frac{2.31}{1.2} = 1.91ma$. The Q2 base voltage then is $\frac{121}{120} * 1.91 + 0.7 = 1.13v$. The results of this (very) approximate calculation are entered in row 1 of the table below.

Neglecting the Q2 base current the Q1 collector current would be 1.08ma. The feedback correction makes this current estimate an improvement over the assumption of zero current. Suppose then the calculation is repeated, except this time assume the Q1 emitter current is $1.08(121/120) = 1.07ma$. The current in the feedback resistor then is $2.1 - 1.07 = 1.03ma$. The second row summarizes the calculations. On this round the estimate for the Q1 emitter current is 1.07ma. A third round is summarized in the fourth row of the table; the next estimate for the Q1 collector current is 1.06 ma. At this point there has been less than a 1% change, and sufficient convergence has been obtained. A PSpice computation (nonlinear transistor models) gives the Q1 collector current as 1.0ma and the Q2 collector current as 6.83ma.

<table>
<thead>
<tr>
<th>I(Q1)</th>
<th>I(Q2)</th>
<th>I(RF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.31</td>
<td>2.1</td>
</tr>
<tr>
<td>1.09</td>
<td>6.31</td>
<td>1.01</td>
</tr>
<tr>
<td>1.07</td>
<td>6.27</td>
<td>1.03</td>
</tr>
<tr>
<td>1.06</td>
<td>6.25</td>
<td>1.04</td>
</tr>
<tr>
<td>1.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The incremental parameter circuit for the amplifier ‘without feedback’ (i.e., include $h_i$, $h_o$, neglect $h_f$, and set $h_r = 0$) is drawn below.

The voltage gain is readily calculated after transforming the emitter resistors into the respective base circuits, and is

$$G_{NFB}^V = \frac{1}{10 + 100} \frac{1}{[2.96 + (121)(0.1)]} \frac{100}{100 + 2.96 + (121)(0.1)} (-120) \times \frac{8.2}{8.2 + (0.5 + 12.1)} (-120) (1.2) \times (1.1)$$

$$= 128.5$$
Apply the feedback formula to calculate the voltage gain with feedback:

\[ G_{FB}^y = \frac{128.5}{1 + (128.5/10)} = 9.28 \ (19.35 \text{ dB}) \]

PSpice, using nonlinear transistor models, computes a gain of 18.5 db

4) Each amplifier icon in the amplifier circuit drawn to the right represents a device with an input resistance of 1KΩ and a current gain of 50; the output resistance and the feedback are zero.

Calculate the overall current gain.

The equivalent circuit obtained on substituting the amplifier circuit is:

Note that the first stage has local shunt-shunt (‘y’ parameter) feedback, for which the ‘natural’ gain is a transimpedance. The second stage has local series-series (‘z’ parameter) feedback, for which the ‘natural’ gain is a transadmittance. The overall gain (product) is a current gain. If the amplifiers around which the feedback is applied are regarded as idealized the current gain can be estimated as \(-10/0.47 = 21.3\). However as will be seen the feedback in this case adds significant loading to the amplifiers to make them less than idealized.

To simplify the analysis a bit transform the second stage load into the base as shown in the upper circuit diagram to the right the output current is simply related to \(i_{b2}\). The lower diagram is the amplifier ‘without feedback’, i.e., it accounts for the loading of \(y_i\) and \(y_o\), neglects \(y_f\) compared to the active amplifier, and omits \(y_r\).

The transimpedance gain ‘without feedback’ is

\[ G_{NFB}^Z = \frac{4.7\|10}{1 + (4.7\|10)} \ (-50) \ (0.47\|10\|24.97) \]

\[ = -16.8K \]
and the gain 'with feedback' is

\[ G_{FB} = \frac{-16.8K}{1 + (16.8/10)} = -6.27K \]

To determine the overall current gain calculate \( ib2 \) and multiply by 50;

\[ G_I = (6.27/24.97)(50) = -12.55 \]

5) Each amplifier icon in the amplifier circuit drawn to the right represents a device with an input resistance of 1KΩ and a current gain of 50; the output resistance and the feedback are zero. See Problem 4) for the equivalent circuit. Consider the amplifier to have series-shunt feedback as indicated, and calculate the overall voltage gain.

The amplifier symbols are replaced in the circuit below by explicit circuit elements. Note that what is involved is no more than simplified transistor models. For convenience the netlist converts the current-controlled current sources into voltage-controlled current sources.

PSpice computes a voltage gain of 17.4 (24.8 dB). Note that an estimate of the voltage gain assuming the feedback is applied about an idealized amplifier is \((10 + 0.47)/0.47 = 22.3\)

Consider this to be series-shunt feedback. The amplifier 'without feedback' is obtained as described above by transforming the feedback circuit into its hybrid-parameter equivalent, removing the \( h_r \) term, and neglecting the \( h_f \) term; the circuit is drawn next. Note that what is involved is not simply a matter of opening the feedback loop by removing the 10 KΩ resistor. The input and output loading effect of this resistor have been included, \( h_r \) has been accounted for (neglected) explicitly, and \( h_f \) will be accounted for as part of the formalism.
6) Calculate the voltage gain for the complementary-pair amplifier circuit drawn to the right. Compare the calculated gain with a PSpice computation.

$$G_{NFB}^V = \frac{1}{4.7 + 1 + (50)(0.47)(10)} \cdot \frac{(-50)(-50)(1||10.47)}{79.81} = 79.81$$

$$G_{FB}^V = \frac{79.81}{1 + (79.81)(0.47/10.47)} = 17.42$$
Except that Q1 is assumed not to be saturated (to be verified) an estimate of the Q1 emitter current using the simplified PWL model does not involve the collector. The PSpice computed collector current is 0.75 ma.

Neglect the Q2 base current (compared to the Q1 collector current) to estimate the Q2 collector current, and from this the Q2 emitter current. The PSpice computed collector current is 0.657 ma.

**NODE VOLTAGES (PSpice)**

(1) 0.0000  (2) 2.4054  (3) 1.7493  (4) 9.0666  
(5) 12.0000  (6) 9.8220  (7) 9.7561  (8) 4.4679  (9) 1.6732

\[ \text{IE1} = \frac{10}{10 + 39} \times \frac{12 - 0.7}{2.3 + 10} = 0.74 \text{ ma} \]

\[ \text{rbe1} = \frac{1.21}{0.74} = 5.3 \text{ K} \]

\[ \text{IE2} = \frac{(0.74)(3.9) - 0.7}{5.4} = 0.64 \text{ ma} \]

\[ \text{rbe2} = \frac{1.21}{0.64} = 4.9 \text{ K} \]

\[ G^V = \frac{1}{10 \times (39 + (5.3 + 12.1))} \times (-120) \times \frac{3.9}{3.9 + 4.9 + 12.1} \times (-120) \times 6.81 \]

\[ = 829.3 \text{ (58.4 db)} \]

**PSpice computes a voltage gain of 58.4 db**

7) An amplifier triangle symbol is used, as shown, to represent an incremental model with a 1Ω (scaled) input resistance and an incremental current gain of 50. Use two-port feedback theory to estimate the transfer ratios for the circuits shown.
Interpret this as series-shunt ('h' parameter) feedback, identifying the feedback as shown below. If the amplifier (without feedback is regarded as idealized) the gain with feedback is estimated as $100/0.5 = 20$. A more detailed calculation is shown to the right.

'Feedback' is in part a point of view rather than a specific circuit configuration. For this circuit, for example, the feedback can be interpreted as the shunt-series connection pictured below. The 'output' for this interpretation is $i_2$, although the problem actually references the output current $50i_2$. The natural gain is the current gain (emphasizing however that the output current for applying the formalism is $i_2$).
The circuit 'without feedback is shown below (see 'g' parameter discussion).

\[ i_1 = \frac{5 \parallel 10.5}{1 + (10.5 \parallel 5)} = 0.7721 \]

Note that the voltage drop across this branch is the same as for the branch in which \( i_2 \) flows.

Hence \[ 50i_1 = 51i_2 + (1 + 0.5 \parallel 10)i_2 \]
\[ = 52.472i_2 \]

\[ i_2 = 0.7357 \] This is the current gain 'without' feedback

\[ i_2 = \frac{0.7357}{1 + (0.7357)(0.5/10.5)} = 0.71 \] (with feedback)

The problem actually asks for \( 50i_2 = 35.5 \)
\[
\begin{align*}
\begin{bmatrix}
-1.65 \\
0
\end{bmatrix}
&= 
\begin{bmatrix}
\frac{1}{6.1} + \frac{1}{10} & -\frac{1}{10} \\
\frac{50}{6.1} - \frac{1}{10} & \frac{1}{10} + \frac{1}{110.5}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
= 
\begin{bmatrix}
0.2639 & -0.1 \\
6.0967 & 1.2
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}

\Rightarrow e_2 = 11.87
\]