8.F. SUPPLEMENTAL TOPIC: ML-BASED AND LR-BASED LOWER 100(\text{scp})\% (ONE-SIDED) ASYMPTOTIC STATISTICAL CONFIDENCE BANDS AND LIMITS

ML-Based Lower 100(\text{scp})\% (One-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF for Life (Endurance) Data and for Strength (Resistance) Data:
Recall that the 100(\text{scp})\% joint confidence region that allegedly contains the actual values of the \text{clp0} and the \text{clp1} was used in simple linear regression to compute hyperbolic 100(\text{scp})\% (two-sided) statistical confidence bands that allegedly include mean(\text{APRCRHNDRDV's}) for all values of the \text{ivv} employed in the regression experiment test program. Analogously, ML-based 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands that allegedly bound the actual values for all percentiles of the presumed conceptual two-parameter distribution can be computed using the 100(\text{scp})\% joint asymptotic statistical confidence region that allegedly contains the actual values for the \text{clp} and the \text{csp}. Then, these 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands will be re-interpreted as lower 100[(1+\text{scp})/2]\% (one-sided) asymptotic statistical confidence (tolerance) bands.

Recall also that the elliptical boundary of the 100(\text{scp})\% joint statistical confidence region that allegedly contains the actual values for the \text{clp0} and the \text{clp1} in simple linear regression is uniquely established by a \text{scp}-based selected value for Snedecor’s central F statistic with 2 numerator and (n_{rdr}-2) denominator statistical degrees of freedom (Supplemental Topic 7.C). The analogous elliptical 100(\text{scp})\% joint statistical confidence region that allegedly includes the actual values for the \text{clp} and the \text{csp} is asymptotic in ML analysis, viz., its actual probability content only asymptotically approaches the selected \text{scp} value. Accordingly, the number of denominator statistical degrees of freedom for the analogous Snedecor’s central F statistic becomes infinite. Then, as indicated in Figure 5.7, it can be re-expressed as Pearson’s central \chi^2 statistic with 2 (numerator) statistical degrees of freedom. In turn, the ML-based joint 100(\text{scp})\% asymptotic statistical confidence region that allegedly includes the actual values for the \text{clp} and the \text{csp} is established by the quadratic expression

\[
\chi^2_{2,\text{scp}} = -\frac{\partial^2 \log_e(\text{likelihood})}{\partial \text{clp}^2} \left[ \text{est}(\text{clp}) - \text{clp}_{\text{critical}} \right]^2
- \frac{\partial^2 \log_e(\text{likelihood})}{\partial \text{csp}^2} \left[ \text{est}(\text{csp}) - \text{csp}_{\text{critical}} \right]^2
- 2 \frac{\partial^2 \log_e(\text{likelihood})}{\partial \text{clp} \partial \text{csp}} \left[ \text{est}(\text{clp}) - \text{clp}_{\text{critical}} \right] \left[ \text{est}(\text{csp}) - \text{csp}_{\text{critical}} \right]
\]

in which the partial derivatives must be evaluated at est(\text{clp}) and est(\text{csp}). Note that this ML-based joint 100(\text{scp})\% asymptotic statistical confidence region is elliptical regardless of the parameterization of the presumed conceptual two-parameter distribution.

The corresponding LR-based joint 100(\text{scp})\% asymptotic statistical confidence region is not elliptical. Rather, its boundary is established by the trace on the \log_e (likelihood) surface at which the magnitude of the corresponding \log_e (likelihood) is less than \log_e (maximized likelihood) by the value (1/2) \chi^2_{2,\text{scp}}. This boundary depends markedly on which of the following parameterizations is employed in ML analysis:

1. \( y = \text{csp} \cdot [\log_e(fnc) - \text{clp}] \)
2. \( y = \text{clp} + \text{csp} \cdot [\log_e(fnc)] \)
3. \( y = \frac{\log_e(fnc) - \text{clp}}{\text{csp}} \)
or,

\[
(4) \quad y = \frac{\log_2(\text{fnc})}{\text{csp}} - \text{clp}
\]

Nevertheless, the associated LR-based 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands are all the same, viz., do not depend on the specific parameterization.

All joint 100(\text{scp})\% asymptotic statistical confidence regions can be depicted on \text{clp},\text{csp} co-ordinates by plotting numerous points that lie on their respective boundaries. These boundary points pertain to coupled values of \text{clp}_{\text{critical}} and \text{csp}_{\text{critical}} for which the null hypothesis that these hypothetical values for the \text{clp} and the \text{csp} are correct will just be rejected when the acceptable probability of committing a Type I error is equal to (1-\text{scp}). In our microcomputer programs 3600 coupled boundary point values for \text{clp}_{\text{critical}} and \text{csp}_{\text{critical}} are computed, one boundary point value for each one-tenth of a degree of counterclockwise rotation around the ML-estimated center of the 100(\text{scp})\% joint asymptotic statistical confidence region. These 3600 coupled values of \text{clp}_{\text{critical}} and \text{csp}_{\text{critical}} are used to compute the 3600 corresponding hypothetical CDF’s. Then, for each \text{y} value or metric value of specific interest, the respective minimum and maximum [est(metric value given the \text{y} of specific interest)]’s or [est(\text{y} value given the metric value of specific interest)]’s associated with these 3600 hypothetical CDF’s establish reasonably accurate numerical values for the 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands that allegedly include the actual CDF. When \text{y} pertains to the value of \text{pf} of specific interest, the respective minimums of the 3600 [est(metric value given the \text{y} of specific interest)]’s are ML-based and LR-based lower 100[(1+\text{scp})/2]\% (one-sided) asymptotic statistical tolerance limits.

The respective maximums and minimums of the 3600 computed hypothetical [est(metric value given the \text{y} of specific interest)]’s do not correspond one-to-one to the associated maximums and minimums of the 3600 [est(\text{y} value given the metric value of specific interest)]’s, or vice versa, for any of the four parameterizations considered herein. Moreover, only the two linear parameterizations, viz., (2) and (3), have their ML-based elliptical joint 100(\text{scp})\% asymptotic statistical confidence region bisected by the straight line that connects the hypothetical [\text{clp},\text{csp}] points associated with the respective maximums and minimums of the 3600 computed values of the [est(metric value given the \text{y} of specific interest)]’s and [est(\text{y} value given the metric value of specific interest)]’s. Accordingly, only linear parameterizations (2) and (3) generate hyperbolic 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands such that the probability that the actual CDF lies (in part) above the upper band exactly equal to the probability that the actual CDF lies (in part) below the lower band, viz., both probabilities are equal to (1-\text{scp})/2, regardless of the value for \text{n}_{\text{dv}}. This probability behavior is only asymptotically approached for parameterizations (1) and (4), and for LR-based 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands.

(a) **Life (Endurance) Data:** Recall that we consider two alternative statistical models for the mechanical reliability life (endurance) of specific interest, viz., either a conceptual (two-parameter) smallest-extreme-value distribution or a conceptual (two-parameter) normal distribution, each having a logarithmic fnc metric. The LSEV\(J\)BAC and LN\(J\)BACN series of microcomputer programs compute points on the lower band of ML-based and LR-based 100(\text{scp})\% (two-sided) asymptotic statistical confidence bands. These points are then used to compute ML-based and LR-based 100(\text{scp})\% (two-sided) asymptotic statistical tolerance limit bands that allegedly bound the actual CDF for all values of its logarithmic metric. In turn, given either the value for fnc* of specific interest or the CDF percentile of specific interest, the logarithmic metric value can be computed for the corresponding point lying on the lower of the two asymptotic statistical tolerance limit bands. This logarithmic metric value is an asymptotic A-basis statistical tolerance limit when \text{scp} is equal to 0.90 and the CDF percentile of specific interest is its first, viz., \text{pf} = 0.01. Two sets of four alternative asymptotic A-basis statistical tolerance
limits computed by running the $LSEV(J)BAC$ and $LN(J)BACN$ series of microcomputer programs are given in Table 8F.1 for purposes of comparison. Note that the respective exemplar asymptotic statistical tolerance limits differ markedly. However, the differences among these alternative asymptotic statistical tolerance limits will decrease as $n_{dp}$ increases. This behavior is evident in Table 8F.2, where it is also evident that these differences are markedly decreased by employing the pragmatic bias-correction methodology in Supplemental Topic 8.E.

**Table 8F.1.** Respective ML-based and LR-based $A$-basis Statistical Tolerance Limits computed by running the $LSEV(J)BAC$ and $LN(J)BACN$ series of microcomputer programs with the input datum values that appear in microcomputer files $WBLDTAAC$ and $LNDATAAC$.

<table>
<thead>
<tr>
<th>CDF Parameterization</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptual Two-Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Weibull Distribution</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower (One-Sided) Asymptotic Statistical Confidence Limit Computed by Running the $LSEV(J)AAC$ Series of Microcomputer Programs (Method Two)</td>
<td></td>
<td></td>
<td>34.584</td>
<td></td>
</tr>
<tr>
<td>Computed Using the Joint Asymptotic Confidence Region</td>
<td>4.331</td>
<td>3.901</td>
<td>71.811</td>
<td>1.679</td>
</tr>
<tr>
<td>Computed Using the Likelihood Ratio Method</td>
<td>32.254</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conceptual Two-Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log$_e$-Normal Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower (One-Sided) Asymptotic Statistical Confidence Limit Computed by Running the $LN(J)AACN$ Series of Microcomputer Programs (Method Two)</td>
<td></td>
<td></td>
<td></td>
<td>112.714</td>
</tr>
<tr>
<td>Computed Using the Joint Asymptotic Confidence Region</td>
<td>57.493</td>
<td>57.357</td>
<td>143.704</td>
<td>7.625</td>
</tr>
<tr>
<td>Computed Using the Likelihood Ratio Method</td>
<td>92.828</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 8F.2.** Respective ML-based and LR-based $A$-basis statistical tolerance limits computed by running the $LN(J)ANCN$ series of microcomputer programs with the 21 input datum values that appear in microcomputer file $STLDATA$.

<table>
<thead>
<tr>
<th>CDF Parameterization</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptual Two-Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log$_e$-Normal Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed by Running Microcomputer Program $ABLNSTLN$ (for Reference)</td>
<td></td>
<td></td>
<td>114304</td>
<td></td>
</tr>
<tr>
<td>Lower (One-Sided) Asymptotic Statistical Confidence Limit Computed by Running the $LN(J)ANCN$ Series of Microcomputer Programs (Method Two)</td>
<td></td>
<td></td>
<td>114709</td>
<td></td>
</tr>
<tr>
<td>Associated Pragmatic Bias-Corrected $A$-basis Statistical Tolerance Limit Based on 30000 &quot;Replicate&quot; Pseudorandom Data Sets</td>
<td></td>
<td></td>
<td>114406</td>
<td></td>
</tr>
<tr>
<td>Computed Using the Joint Asymptotic Statistical Confidence Region</td>
<td>113663</td>
<td>113447</td>
<td>114679</td>
<td>116010</td>
</tr>
<tr>
<td>Associated Pragmatic Bias-Corrected $A$-basis Statistical Tolerance Limit Based on 30000 &quot;Replicate&quot; Pseudorandom Data Sets</td>
<td>114164</td>
<td>114103</td>
<td>114083</td>
<td>114712</td>
</tr>
<tr>
<td>Computed Using the Likelihood Ratio Method</td>
<td>113723</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Associated Pragmatic Bias-Corrected $A$-basis Statistical Tolerance Limit Based on 30000 &quot;Replicate&quot; Pseudorandom Data Sets</td>
<td></td>
<td></td>
<td>114134</td>
<td></td>
</tr>
</tbody>
</table>
Given actual Type I censoring and presuming a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = \text{csp} \cdot [\log_e(fnc) - clp] \]

and with no statistical bias corrections

\[
\begin{align*}
est(clp) &= 0.6061362736D+01 \\
est(csp) &= 0.4731943406D+01 \\
est\{\text{var}\{est(clp)\}\} &= 0.9059101593D-02 \\
est\{\text{var}\{est(csp)\}\} &= 0.3013638227D+01 \\
est\{\text{covar}\{est(clp),est(csp)\}\} &= 0.1956788216D-01 \\
est(\text{conceptual correlation coefficient}) &= 0.1184283562D+00
\end{align*}
\]

<table>
<thead>
<tr>
<th>(fnc)</th>
<th>(est(y))</th>
<th>(est(pf))</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-2.0694929</td>
<td>.1186053</td>
</tr>
<tr>
<td>310.000</td>
<td>-1.5368900</td>
<td>.1934980</td>
</tr>
<tr>
<td>374.000</td>
<td>-0.6487823</td>
<td>.4070717</td>
</tr>
<tr>
<td>402.000</td>
<td>-0.3071535</td>
<td>.5207523</td>
</tr>
<tr>
<td>456.000</td>
<td>0.2892640</td>
<td>.7369587</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(snc)</th>
<th>(est(y))</th>
<th>(est(pf))</th>
</tr>
</thead>
<tbody>
<tr>
<td>500.000</td>
<td>0.7251484</td>
<td>0.8731865</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \(clp\) and the \(csp\) (with 2 Statistical Degrees of Freedom)

Lower Boundary Point = at \(fnc\) =

53.438 0.377 000 310.000
90.623 0.374 000 402.000
217.892 0.456 000
302.833 456.000
371.903

at \(snc\) =

403.022 500.000

Lower Boundary Point = est(\(fnc\)) = 4.331 162.262

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point = at \(fnc\) =

126.067 277.000
165.957 310.000
255.244 374.000
296.154 402.000
369.401 456.000

at \(snc\) =

417.964 500.000

Lower Boundary Point = est(\(fnc\)) = 32.254 162.262
COPY WBLDTAAC DATA
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LSEV2BAC

Given actual Type I censoring and presuming a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest extreme-value distribution in ML analysis with the parameterization

\[ y = clp + csp \cdot \log(y) \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= -2868202543D+02 \\
\text{est}(csp) &= .473943406D+01 \\
\text{est}\{\text{var}\{\text{est}(clp)\}\} &= .1120467627D+03 \\
\text{est}\{\text{var}\{\text{est}(csp)\}\} &= .3013638227D+01 \\
\text{est}\{\text{covar}\{\text{est}(clp),\text{est}(csp)\}\} &= -.1835934856D+02 \\
\text{est}(\text{conceptual correlation coefficient}) &= -.9991071169D+00
\end{align*}
\]

\[
\begin{array}{ccc}
\text{fnc} & \text{est}(y) & \text{est}(pf) \\
277.000 & -2.0694929 & .1186053 \\
310.000 & -1.5368900 & .1934980 \\
374.000 & -.6487823 & .4007017 \\
402.000 & -.3071535 & .5207523 \\
456.000 & .2892640 & .7369587 \\
\text{snc} & \text{est}(y) & \text{est}(pf) \\
500.000 & .7251484 & .8731865
\end{array}
\]

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

\[
\begin{align*}
\text{at } fnc &= \text{at } snc \\
46.425 & \text{ at } fnc = 277.000 \\
77.873 & \text{ at } snc = 310.000 \\
176.313 & \text{ at } fnc = 374.000 \\
234.472 & \text{ at } snc = 402.000 \\
342.820 & \text{ at } fnc = 456.000
\end{align*}
\]

Lower Boundary Point =

\[
\begin{align*}
\text{at } fnc(01) &= \text{est}(fnc) = 3.901 \\
\text{at } snc &= 162.262
\end{align*}
\]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point =

\[
\begin{align*}
\text{at } fnc &= \text{at } snc \\
126.062 & \text{ at } fnc = 277.000 \\
165.957 & \text{ at } snc = 310.000 \\
255.244 & \text{ at } fnc = 374.000 \\
296.154 & \text{ at } snc = 402.000 \\
369.401 & \text{ at } fnc = 456.000
\end{align*}
\]

Lower Boundary Point =

\[
\begin{align*}
\text{at } fnc(01) &= \text{est}(fnc) = 417.964 \\
\text{at } snc &= 500.000 \\
32.254 & \text{ at } fnc(01) = 162.262
\end{align*}
\]
Given actual Type I censoring and presuming a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = \log_e(fnc) - \frac{clp}{csp} \]

and with no statistical bias corrections

\[
\begin{align*}
est(clp) &= 0.6061362736 \times 10^0 \\
est(csp) &= 0.2113296619 \times 10^0 \\
est[var(est(clp))] &= 0.9059101593 \times 10^{-2} \\
est[var(est(csp))] &= 0.6010809287 \times 10^{-2} \\
est[covar(est(clp),est(csp))] &= -0.8739063943 \times 10^{-3} \\
est(conceptual correlation coefficient) &= -0.1184283562 \times 10^0
\end{align*}
\]

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

\[
\begin{align*}
fnc &= 181.888 \\
219.406 \\
293.594 \\
323.796 \\
371.768 \\
\end{align*}
\]

at fnc =

\[
\begin{align*}
fnc &= 181.888 \\
219.406 \\
293.594 \\
323.796 \\
371.768 \\
\end{align*}
\]

at fnc(01) =

\[
\begin{align*}
71.811 \\
162.262
\end{align*}
\]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point =

\[
\begin{align*}
fnc &= 126.062 \\
165.957 \\
255.244 \\
296.154 \\
369.401 \\
\end{align*}
\]

at fnc =

\[
\begin{align*}
fnc &= 126.062 \\
165.957 \\
255.244 \\
296.154 \\
369.401 \\
\end{align*}
\]

at fnc(01) =

\[
\begin{align*}
417.964 \\
500.000 \\
\end{align*}
\]

at fnc(01) =

\[
\begin{align*}
32.254 \\
162.262
\end{align*}
\]
\texttt{COPY WEBDATA2C DATA}
\texttt{1 file(s) copied}

\texttt{LSEV4BAC}

Given actual \emph{Type I} censoring and presuming a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = \frac{\log(fnc)}{csp} - clp \]

and with no statistical bias corrections

\[
\begin{array}{ccc}
\text{est}(clp) = & 0.2868202543D+02 & \text{est}(csp) = 0.2113296619D+00 \\
\text{est}([\text{var(est}(clp)]) = & 0.1120467627D+03 & \text{est}([\text{var(est}(csp)]) = 0.6010809287D-02 \\
\text{est}([\text{covar(est}(clp),est}(csp)]) = & -0.8199326557D+00 & \text{est(}\text{conce}ntual\text{ correlation coefficient)} = -0.9991071169D+00 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{fnc} & \text{est}(y) & \text{est}(\rho f) \\
277.000 & -2.0694929 & 0.1186053 \\
310.000 & -1.5368900 & 0.1934980 \\
374.000 & -0.6487823 & 0.4070717 \\
402.000 & -0.3071535 & 0.5207523 \\
456.000 & 0.2892640 & 0.7369587 \\
\end{array}
\]

\[
\begin{array}{ccc}
nesc & \text{est}(y) & \text{est}(\rho f) \\
500.000 & 0.7251484 & 0.8731865 \\
\end{array}
\]

Points on the Lower Band of the 90\% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \text{clp} and the \text{csp} (with 2 Statistical Degrees of Freedom)

\[
\begin{array}{c}
\text{Lower Boundary Point} = \\
4.365 \\
5.338 \\
7.465 \\
8.493 \\
10.229 \\
\end{array}
\]

\[
\begin{array}{c}
at \text{fnc} = \\
277.000 \\
310.000 \\
374.000 \\
402.000 \\
456.000 \\
\end{array}
\]

\[
\begin{array}{c}
10.432 \\
500.000 \\
\end{array}
\]

\[
\begin{array}{c}
at \text{nesc} = \\
\end{array}
\]

\[
\begin{array}{c}
\text{Lower Boundary Point} = \\
1.679 \\
162.262 \\
\end{array}
\]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

\[
\begin{array}{c}
\text{Lower Boundary Point} = \\
126.067 \\
165.957 \\
255.244 \\
296.154 \\
369.401 \\
\end{array}
\]

\[
\begin{array}{c}
at \text{fnc} = \\
277.000 \\
310.000 \\
374.000 \\
402.000 \\
456.000 \\
\end{array}
\]

\[
\begin{array}{c}
at \text{nesc} = \\
417.964 \\
500.000 \\
\end{array}
\]

\[
\begin{array}{c}
\text{Lower Boundary Point} = \\
32.254 \\
162.262 \\
\end{array}
\]
Given actual Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = csp \cdot [\log(c) - clp] \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= .5956793964D+01 \quad \text{est}(csp) = .4171442213D+01 \\
\text{est} \{\text{var}[\text{est}(clp)]\} &= .9987394347D-02 \quad \text{est} \{\text{var}[\text{est}(csp)]\} = .1903271765D+01 \\
\text{est} \{\text{covar}[\text{est}(clp),\text{est}(csp)]\} &= -.1346739280D-01 \quad \text{est} \{\text{conceputal correlation coefficient}\} = -.9768030920D-01
\end{align*}
\]

<table>
<thead>
<tr>
<th>fnC</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
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<td>277.000</td>
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<td>.0825445</td>
</tr>
<tr>
<td>310.000</td>
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<td>.1791414</td>
</tr>
<tr>
<td>374.000</td>
<td>-.1357311</td>
<td>.4460169</td>
</tr>
<tr>
<td>402.000</td>
<td>.1654316</td>
<td>.5656978</td>
</tr>
<tr>
<td>456.000</td>
<td>.6912032</td>
<td>.7552811</td>
</tr>
<tr>
<td>500.000</td>
<td>1.0754568</td>
<td>.8589149</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands
that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region
for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

\[
\begin{align*}
&\text{at fnC} = \\
&124.351 \quad 277.000 \\
&182.454 \quad 310.000 \\
&302.115 \quad 374.000 \\
&323.179 \quad 402.000 \\
&358.474 \quad 456.000 \\
&\text{at snc} = \\
&384.429 \quad 500.000
\end{align*}
\]

Lower Boundary Point =

\[
\text{at fnC(01)}
\]

57.493 \quad 221.209

Computed Using 3600 Points on the Boundary of the Corresponding Region
Established by the Likelihood Ratio Method

Lower Boundary Point =

\[
\begin{align*}
&\text{at fnC} = \\
&155.627 \quad 277.000 \\
&198.266 \quad 310.000 \\
&282.348 \quad 374.000 \\
&315.504 \quad 402.000 \\
&368.378 \quad 456.000 \\
&\text{at snc} = \\
&401.906 \quad 500.000 \\
&\text{at fnC(01)}
\end{align*}
\]

Lower Boundary Point =

\[
\text{est(fnC)} = \\
92.828 \quad 221.209
\]
Given actual Type I censoring and presuming a conceptual two-parameter log-$e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

$$y = clp + csp \cdot \log_e(fnc)$$

and with no statistical bias corrections

<table>
<thead>
<tr>
<th>$fnc$</th>
<th>$est(y)$</th>
<th>$est(pf)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3881578</td>
<td>0.0825445</td>
</tr>
<tr>
<td>310.000</td>
<td>-0.9186420</td>
<td>0.1791414</td>
</tr>
<tr>
<td>374.000</td>
<td>-0.1357311</td>
<td>0.4460169</td>
</tr>
<tr>
<td>402.000</td>
<td>0.1654316</td>
<td>0.5656978</td>
</tr>
<tr>
<td>456.000</td>
<td>0.6912032</td>
<td>0.7552811</td>
</tr>
</tbody>
</table>

$sec$

<table>
<thead>
<tr>
<th>$est(y)$</th>
<th>$est(pf)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500.000</td>
<td>1.0754568</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the $clp$ and the $csp$ (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

| 120.103 | 277.000 |
| 169.937 | 310.000 |
| 275.414 | 374.000 |
| 313.548 | 402.000 |
| 368.208 | 456.000 |

at $fnc$ =

| 277.000 |
| 310.000 |
| 374.000 |
| 402.000 |
| 456.000 |

Lower Boundary Point =

| 401.704 |
| 500.000 |

at $fnc(01)$

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point =

| 155.627 | 277.000 |
| 198.266 | 310.000 |
| 282.348 | 374.000 |
| 315.504 | 402.000 |
| 368.378 | 456.000 |

at $fnc$ =

| 277.000 |
| 310.000 |
| 374.000 |
| 402.000 |
| 456.000 |

Lower Boundary Point =

| 401.906 |
| 500.000 |

at $fnc(01)$

| 221.209 |
Given actual Type I censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \left( \log_e(fnc) - c/l[p] \right) / csp \]

and with no statistical bias corrections

\[
\begin{align*}
est(c/l[p]) &= .5959062671D+01 & \est(csp) &= .2412998143D+00 \\
est\{\text{var}[c/l[p]]\} &= .9921960558D-02 & \est\{\text{var}[csp]\} &= .6305038312D-02 \\
est\{\text{covar}[c/l[p],csp]\} &= .6974504622D-03 & \est(\text{conceptual correlation coefficient}) &= .8818013378D-01
\end{align*}
\]

<table>
<thead>
<tr>
<th>\text{fnc}</th>
<th>\text{est}(y)</th>
<th>\text{est}(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3881578</td>
<td>.0825445</td>
</tr>
<tr>
<td>310.000</td>
<td>-.9186420</td>
<td>.1791414</td>
</tr>
<tr>
<td>374.000</td>
<td>-.1357311</td>
<td>.4460169</td>
</tr>
<tr>
<td>402.000</td>
<td>.1654316</td>
<td>.5656978</td>
</tr>
<tr>
<td>456.000</td>
<td>.6912032</td>
<td>.7552811</td>
</tr>
<tr>
<td>500.000</td>
<td>1.0754568</td>
<td>.8589149</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90\% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( c/l[p] \) and the \( csp \) (with 2 Statistical Degrees of Freedom)

Lower Boundary Point = at \( fnc = \)

\[
\begin{align*}
204.569 & \quad 277.000 \\
240.769 & \quad 310.000 \\
302.115 & \quad 374.000 \\
322.932 & \quad 402.000 \\
353.552 & \quad 456.000
\end{align*}
\]

at \( snc = \)

\[
372.193 \quad 500.000
\]

Lower Boundary Point = \( \est(fnc) = \)

\[
143.704 \quad 221.209
\]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point = at \( fnc = \)

\[
\begin{align*}
155.627 & \quad 277.000 \\
198.266 & \quad 310.000 \\
282.348 & \quad 374.000 \\
315.504 & \quad 402.000 \\
368.378 & \quad 456.000
\end{align*}
\]

at \( snc = \)

\[
401.906 \quad 500.000
\]

Lower Boundary Point = \( \est(fnc) = \)

\[
92.828 \quad 221.209\]
Given actual Type I censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \{[\text{loge}(fnc)]/csp\} - \text{clp} \]

and with no statistical bias corrections:

<table>
<thead>
<tr>
<th>clp</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>est(clp)</td>
<td>= -2484842180D+02</td>
<td></td>
</tr>
<tr>
<td>est[covar(est(clp))]</td>
<td>= .6703904559D+02</td>
<td></td>
</tr>
<tr>
<td>est[covar(est(clp),est(csp))]</td>
<td>= -6483112811D+02</td>
<td></td>
</tr>
<tr>
<td>est(est(csp))</td>
<td>= .2397252434D+00</td>
<td></td>
</tr>
<tr>
<td>est[est(csp)]</td>
<td>= .6285732256D-02</td>
<td></td>
</tr>
<tr>
<td>est(conceptual correlation coefficient)</td>
<td>= -9987153578D+00</td>
<td></td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF:

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom):  

Lower Boundary Point:  
\begin{align*}
\text{at}\ fnc &= \\
11.200 & 277.000 \\
13.577 & 310.000 \\
18.703 & 374.000 \\
19.390 & 402.000 \\
20.114 & 456.000 \\
\end{align*}

\text{at}\ snc = \\
\begin{align*}
20.660 & 500.000 \\
\end{align*}

Lower Boundary Point:  
\begin{align*}
\text{at}\ fnc &= \\
7.625 & 221.209 \\
\end{align*}

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method:

Lower Boundary Point:  
\begin{align*}
\text{at}\ fnc &= \\
155.627 & 277.000 \\
198.266 & 310.000 \\
282.348 & 374.000 \\
315.504 & 402.000 \\
368.378 & 456.000 \\
\end{align*}

\text{at}\ snc = \\
\begin{align*}
401.906 & 500.000 \\
\end{align*}

Lower Boundary Point:  
\begin{align*}
\text{at}\ fnc(01) &= \\
92.828 & 221.209 \\
\end{align*}
LR-Based Median Pragmatic Bias-Corrected $A$-Basis and $B$-basis Statistical Tolerance Limits for Life (Endurance) Data: The pragmatic bias-correction methodology presented in Supplemental Topic 8.E can be used in conjunction with the LR method to compute median pragmatic bias-corrected $A$-basis and $B$-basis statistical tolerance limits. We recommend routinely running microcomputer programs PLNTRLR1 or PLNTRLRA2 to generate outputs for comparison to the outputs of microcomputer programs PLNTRLPC1 or PLNTRLAC2 — even though these microcomputer programs take several hours to run. The two sets of microcomputer programs compute median pragmatic bias-corrected $A$-basis and $B$-basis statistical tolerance limits that are almost identical for practical purposes. Nevertheless, we recommend adopting the smaller of the two alternative values as the median pragmatic bias-corrected $A$-basis and $B$-basis statistical tolerance limit of specific interest.

```
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COPY PLNTRLMN
  Given no censoring and presuming a conceptual two-parameter log$_e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

  \[
  y = \log_e[(fc) - clp]/csp
  \]

  and with no statistical bias corrections

  \[
  \text{est}(clp) = 0.5927850884D+01 \quad \text{est}(csp) = 0.1944865059D+00
  \]

  \[
  \text{est}\{\text{var}(clp)\} = 0.6304166830D+02 \quad \text{est}\{\text{var}(csp)\} = 0.3152083415D+02
  \]

  \[
  \text{est}\{\text{covar}(clp, csp)\} = -3.312874752D-16 \quad \text{est(\text{conceptual correlation coefficient})} = -7.4317709971D-14
  \]

  Pragmatic Bias-Corrected Lower 95\% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for $fc$ = (01)

  127.929
```

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95\% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for $fc$ (01), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence Region that Allegedly Includes the Actual Values for the $clp$ and the $csp$

The central 100($p$\%) of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>$p$%</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>120.4</td>
</tr>
<tr>
<td>12.5%</td>
<td>184.8</td>
</tr>
<tr>
<td>102.2</td>
<td>211.2</td>
</tr>
<tr>
<td>5%</td>
<td>238.7</td>
</tr>
<tr>
<td>86.0</td>
<td>238.7</td>
</tr>
<tr>
<td>2.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>76.7</td>
<td>257.5</td>
</tr>
<tr>
<td>0.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>61.0</td>
<td>295.3</td>
</tr>
</tbody>
</table>

Its mean is equal to 154.983 and its median is equal to 150.523
Given no censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ \nu = [\log_e(fnc) - clp]/csp \]

and with Version LS statistical bias-correction factors and the exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
\text{est}(clp) & = .5927850884D+01 & \text{est}(csp) & = .2283744907D+00 \\
\text{est}\{\text{var}[\text{est}(clp)]\} & = .756000195D-02 & \text{est}\{\text{var}[\text{est}(csp)]\} & = .4539000117D-02 \\
\text{est}\{\text{covar}[\text{est}(clp),\text{est}(csp)]\} & = -.4354886956D-16 & \text{est}(\text{conceptual correlation coefficient}) & = -.7431770971D-14
\end{align*}
\]

<table>
<thead>
<tr>
<th>(fnc)</th>
<th>(\text{est}(\nu))</th>
<th>(\text{est}(\nu_F))</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3304173</td>
<td>.0916904</td>
</tr>
<tr>
<td>310.000</td>
<td>-.8375655</td>
<td>.2011374</td>
</tr>
<tr>
<td>374.000</td>
<td>-.0157421</td>
<td>.4937201</td>
</tr>
<tr>
<td>402.000</td>
<td>.3003891</td>
<td>.6180598</td>
</tr>
<tr>
<td>456.000</td>
<td>.8522928</td>
<td>.8029742</td>
</tr>
<tr>
<td>475.000</td>
<td>1.0310430</td>
<td>.8487397</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \(fnc(01)\)

127.631

Based on the \(95^{th}\) Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \(fnc(01)\). Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence regiona that Allegedly Includes the Actual Values for the \(clp\) and the \(csp\)

The central \(100(\rho)\)% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>(\rho)</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.3</td>
<td>163.1</td>
<td>220.6</td>
<td></td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
<td>95%</td>
<td></td>
</tr>
<tr>
<td>81.0</td>
<td>190.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>97.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>65.9</td>
<td>241.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5%</td>
<td>99.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43.7</td>
<td>283.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Its mean is equal to 133.788 and its median is equal to 128.082

Recall that microcomputer program \(ELNLTNC1\) computed the exact (unbiased) \(A\)-basis statistical tolerance limit as 127.663. (The ratio of 128.082 to 127.663 is equal to 1.003.)
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FLNTLRNN

Given no censoring and presuming a conceptual two-parameter log$_e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_e(\text{fnc}) - clp] / csp \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est(clp)} &= .5927850884D+01 \\
\text{est(csp)} &= .1944865059D+00 \\
\text{est[\text{var(est(clp))}]} &= .6304166830D-02 \\
\text{est[\text{var(est(csp))}]} &= .3152083415D-02 \\
\text{est[\text{covar(est(clp),est(csp))}]} &= -.3312874752D-16 \\
\text{est(\text{conceptual correlation coefficient})} &= -.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for fnc(10) 197.625

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for fnc(10), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(\rho)\% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% & \quad 75\% \\
189.5 & \quad 247.8 \\
12.5\% & \quad 87.5\% \\
171.0 & \quad 269.7 \\
5\% & \quad 95\% \\
153.7 & \quad 292.5 \\
2.5\% & \quad 97.5\% \\
143.6 & \quad 308.6 \\
0.5\% & \quad 99.5\% \\
124.8 & \quad 338.4
\end{align*}
\]

Its mean is equal to 219.745 and its median is equal to 217.640
C> COPY LIWTLDTAN DATA
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C> PLNTRLN1

Given no censoring and presuming a conceptual two-parameter logc-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \log_c(fmc) - clp/csp \]

and with Version LS statistical bias-correction factors and the exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
est(clp) &= .5927850884D+01 & \text{est}(csp) &= .2283744907D+00 \\
est[\text{var} \{\text{est}(clp)\}] &= .7560000195D-02 & \text{est}[\text{var} \{\text{est}(csp)\}] &= .4539000117D-02 \\
est[\text{covar} \{\text{est}(clp), \text{est}(csp)\}] &= - .4534886956D-16 & \text{est} \{\text{conceptual correlation coefficient} \} &= -.7431770971D-14
\end{align*}
\]

<table>
<thead>
<tr>
<th>FMC</th>
<th>Est(y)</th>
<th>Est(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3304173</td>
<td>0.916904</td>
</tr>
<tr>
<td>310.000</td>
<td>-.8375655</td>
<td>0.201374</td>
</tr>
<tr>
<td>374.000</td>
<td>-.0157421</td>
<td>.4937201</td>
</tr>
<tr>
<td>402.000</td>
<td>.3003891</td>
<td>.6180598</td>
</tr>
<tr>
<td>456.000</td>
<td>.8522928</td>
<td>.8029742</td>
</tr>
<tr>
<td>475.000</td>
<td>1.0310430</td>
<td>.8487397</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95\% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for FMC(10)
197.639

Based on the 95\% Percentile of the Pragmatic Sampling Distribution Comprised of 30000 "Replicate" Realization Values for the Likelihood Ratio Lower 95\% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for FMC(10). Each "Replicate" Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint

Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(p)\% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% &= 168.2 \\
75\% &= 230.5 \\
12.5\% &= 253.6 \\
5\% &= 280.1 \\
1\% &= 97.5 \\
0.5\% &= 298.2 \\
10\% &= 99.5 \\
&= 332.3 \\
\end{align*}
\]

Its mean is equal to 200.942 and its median is equal to 197.929

Recall that microcomputer program ELNTRNC1 computes the exact (unbiased) B-basis statistical tolerance limit as 197.822. (The ratio of 197.929 to 197.822 is equal to 1.001.)
Given no censoring and assuming a conceptual two-parameter log$_e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_e(f_{nc}) - cl/p]/csp \]

and with no statistical bias corrections

\[
\text{est}(cl/p) = .5927850884D+01 \quad \text{est}(csp) = .1944865059D+00
\]

\[
\text{est} \{\text{var(est}(cl/p))\} = .6304166830D-02 \quad \text{est} \{\text{var(est}(csp))\} = .3152083415D-02
\]

\[
\text{est} \{\text{covar(est}(cl/p),est}(csp))\} = -.3312874752D-16 \quad \text{est(}\text{conceptual correlation coefficient}) = -.7431770971D-14
\]

<table>
<thead>
<tr>
<th>$f_{nc}$</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.5622337</td>
<td>.0591165</td>
</tr>
<tr>
<td>310.000</td>
<td>-.9835057</td>
<td>.1626793</td>
</tr>
<tr>
<td>374.000</td>
<td>-.0184850</td>
<td>.4926260</td>
</tr>
<tr>
<td>402.000</td>
<td>.35227299</td>
<td>.6378545</td>
</tr>
<tr>
<td>456.000</td>
<td>1.0007991</td>
<td>.8415380</td>
</tr>
<tr>
<td>475.000</td>
<td>1.2106954</td>
<td>.8869939</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for $f_{nc}(50)$

312.235

Based on the 95\text{th} Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for $f_{nc}(50)$, Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the $cl/p$ and the $csp$

The central 100($p$)% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>$p$</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>301.1</td>
<td>343.0</td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
<td>287.2</td>
</tr>
<tr>
<td>5%</td>
<td>95%</td>
<td>273.3</td>
</tr>
<tr>
<td>2.5%</td>
<td>97.5%</td>
<td>264.9</td>
</tr>
<tr>
<td>0.5%</td>
<td>99.5%</td>
<td>247.7</td>
</tr>
</tbody>
</table>

Its mean is equal to 322.584 and its median is equal to 321.545
Given no censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_e(fnc) - clp]/csp \]

and with Version LS statistical bias-correction factors and the exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
\text{est}(clp) &= 0.5927850884D+01 & \text{est}(csp) &= 0.228374907D+00 \\
\text{est}(\text{var}(clp)) &= 0.756000195D-02 & \text{est}(\text{var}(csp)) &= 0.4539000117D-02 \\
\text{est}(\text{covar}(clp, csp)) &= -0.4354886956D-16 & \text{est}(\text{conceptual correlation coefficient}) &= -0.7431770971D-14
\end{align*}
\]

<table>
<thead>
<tr>
<th>fnc</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3304173</td>
<td>.0916904</td>
</tr>
<tr>
<td>310.000</td>
<td>-.8375655</td>
<td>.2011374</td>
</tr>
<tr>
<td>374.000</td>
<td>-.0157421</td>
<td>.4937201</td>
</tr>
<tr>
<td>402.000</td>
<td>.3003891</td>
<td>.6180598</td>
</tr>
<tr>
<td>456.000</td>
<td>.8522928</td>
<td>.8029742</td>
</tr>
<tr>
<td>475.000</td>
<td>1.0310430</td>
<td>.8487397</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \( fnc(50) \)

315.555

Based on the 95\(^{th}\) Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( fnc(50) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( clp \) and the \( csp \)

The central 100\((p)\)% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% &= 259.1 \\
30\% &= 375.3 \\
75\% &= 337.8 \\
12.5\% &= 274.5 \\
5\% &= 87.5\% \\
2.5\% &= 95\% \\
259.1 &= 2.5\% \\
249.8 &= 2.5\% \\
231.0 &= 2.5\%
\end{align*}
\]

Its mean is equal to 314.722 and its median is equal to 313.234
Supplemental Topic: ML-based and LR-based Lower 100(scp)% (One-Sided) Asymptotic Statistical Confidence Bands and Limits

Given actual Type I censoring and presuming a conceptual two-parameter log$_e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_e(fnc) - clp]/csp \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= .5956793964D+01 \\
\text{est}(csp) &= .2397252434D+00 \\
\text{est}[\var[\text{est}(clp)]] &= .9987394347D-02 \\
\text{est}[\var[\text{est}(csp)]] &= .6285732256D-02 \\
\text{est}[\cov[\text{est}(clp),\text{est}(csp)]] &= .7739467195D-03 \\
\text{est}(\text{conceptual correlation coefficient}) &= .9768030920D-01
\end{align*}
\]

<table>
<thead>
<tr>
<th>fnc</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.3661578</td>
<td>0.825445</td>
</tr>
<tr>
<td>310.000</td>
<td>-0.9186420</td>
<td>0.1791414</td>
</tr>
<tr>
<td>374.000</td>
<td>-1.3573111</td>
<td>0.4460169</td>
</tr>
<tr>
<td>402.000</td>
<td>-1.6543161</td>
<td>0.5656978</td>
</tr>
<tr>
<td>456.000</td>
<td>0.6912032</td>
<td>0.7552811</td>
</tr>
<tr>
<td>snc</td>
<td>est(y)</td>
<td>est(pf)</td>
</tr>
<tr>
<td>500.000</td>
<td>1.0754568</td>
<td>0.8589149</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for fnc(01) 92.890

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 "Replicate" Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for fnc(01), Each "Replicate" Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(p)% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>75%</td>
</tr>
<tr>
<td>79.2</td>
<td>156.3</td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
</tr>
<tr>
<td>57.2</td>
<td>187.9</td>
</tr>
<tr>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>36.9</td>
<td>221.2</td>
</tr>
<tr>
<td>2.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>26.6</td>
<td>244.4</td>
</tr>
<tr>
<td>.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>12.2</td>
<td>289.1</td>
</tr>
</tbody>
</table>

Its mean is equal to 120.677 and its median is equal to 115.379

This microcomputer program ignored 0 "replicate" pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.1411)

This microcomputer program ignored 0 other "replicate" pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 286 other "replicate" pseudorandom data sets that had it Method Two losascf greater than the pre-determined snc* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of &quot;Replicate&quot; Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12151</td>
</tr>
<tr>
<td>1</td>
<td>12018</td>
</tr>
<tr>
<td>2</td>
<td>4896</td>
</tr>
<tr>
<td>3</td>
<td>1049</td>
</tr>
<tr>
<td>4</td>
<td>158</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>
Given actual Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \left[ \log_f(fnc) - clp \right]/csp \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp) based on \( n_f \)

\[
\begin{align*}
est(clp) &= 0.5956793964D+01 \quad \text{est(csp)} = 0.2925788278D+00 \\
est\{\text{var(est(clp))}\} &= 1.248424293D-01 \quad \text{est}\{\text{var(est(csp))}\} = 9.821456650D-02 \\
est\{\text{covar(est(clp),est(csp))}\} &= 1.081623422D-02 \quad \text{est(\text{conceplual correlation coefficient})} = 9.768030920D-01 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>est((fnc))</th>
<th>est((y))</th>
<th>est((pf))</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.1373908</td>
<td>.1276875</td>
<td></td>
</tr>
<tr>
<td>310.000</td>
<td>-.7526917</td>
<td>.2258176</td>
<td></td>
</tr>
<tr>
<td>374.000</td>
<td>-.0112116</td>
<td>.4557243</td>
<td></td>
</tr>
<tr>
<td>402.000</td>
<td>.1355468</td>
<td>.5539102</td>
<td></td>
</tr>
<tr>
<td>456.000</td>
<td>.5663392</td>
<td>.7144184</td>
<td></td>
</tr>
<tr>
<td>500.000</td>
<td>.8811784</td>
<td>.8108894</td>
<td></td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \( fnc(01) \)

94.753

Based on the 95\(^{th}\) Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( fnc(01) \). Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( clp \) and the \( csp \)

The central 100\(\(p\)\)% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{array}{c}
25\% \\
53.8 \\
12.5\% \\
35.5 \\
3\%
\end{array}
\]

\[
\begin{array}{c}
75\% \\
127.4 \\
87.5\% \\
159.6 \\
95\%
\end{array}
\]

\[
\begin{array}{c}
21.4 \\
91.5 \\
2.5\% \\
221.5 \\
99.5\%
\end{array}
\]

\[
\begin{array}{c}
6.6 \\
272.0
\end{array}
\]

Its mean is equal to 94.757 and its median is equal to 86.794.

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1891)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceplual parameter estimates

This microcomputer program ignored 703 other “replicate” pseudorandom data sets that had its Method Two \( losascl \) greater than the pre-determined \( snc^* \) value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8737</td>
</tr>
<tr>
<td>1</td>
<td>12113</td>
</tr>
<tr>
<td>2</td>
<td>72.37</td>
</tr>
<tr>
<td>3</td>
<td>2166</td>
</tr>
<tr>
<td>4</td>
<td>412</td>
</tr>
<tr>
<td>5</td>
<td>38</td>
</tr>
</tbody>
</table>
Given actual Type I censoring and presuming a conceptual two-parameter log-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_e(\text{fnc}) - \text{clp}] / \text{csp} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic \( \text{est(csp)} \) based on \( n_{\text{total}} \)

\[
\begin{align*}
\text{est(clp)} &= 5.956793964D+01 \\
\text{est(csp)} &= 2.814957784D+00 \\
\text{est} \{ \text{var(est(clp))} \} &= 1.198487322D+01 \\
\text{est} \{ \text{var(est(csp))} \} &= 9.051454449D+02 \\
\text{est} \{ \text{covar(est(clp),est(csp))} \} &= 1.017379384D-02 \\
\text{est(\text{conceptual correlation coefficient})} &= 9.768030920D-01
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \( \text{fnc}(01) \)

\[ 94.488 \]

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( \text{fnc}(01) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( \text{clp} \) and the \( \text{csp} \)

The central 100\( (\rho) \)% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
\text{25\%} & \quad 75\% \\
25\% & \quad 75\% \\
58.3 & \quad 133.2 \\
12.5\% & \quad 87.5\% \\
39.3 & \quad 165.3 \\
5\% & \quad 95\% \\
24.0 & \quad 200.7 \\
2.5\% & \quad 97.5\% \\
16.4 & \quad 226.1 \\
5\% & \quad 99.5\% \\
7.2 & \quad 276.0
\end{align*}
\]

Its mean is equal to 99.611 and its median is equal to 91.901.

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1799)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 591 other “replicate” pseudorandom data sets that had its Method Two \text{loascl} greater than the pre-determined \( \text{snr}^* \) value selected for Type I censoring

\[
\begin{array}{|c|c|}
\hline
\text{Number of Type I Censored Datum Values} & \text{Number of “Replicate” Pseudorandom Data Sets} \\
\hline
0 & 9253 \\
1 & 12105 \\
2 & 6845 \\
3 & 1903 \\
4 & 356 \\
5 & 29 \\
\hline
\end{array}
\]
Given actual Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \log(e^{f_{nc}} - clp)/csp \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= .5956793964D+01 \\
\text{est}(csp) &= .2397252434D+00 \\
\text{est}([\text{var}(\text{est}(clp))] &= .998794347D-02 \\
\text{est}([\text{var}(\text{est}(csp))]) &= .6285732256D-02 \\
\text{est}([\text{var}(\text{est}(clp)),\text{est}(csp)]) &= .7739467195D-03 \\
\text{est}([\text{var}(\text{est}(csp))]) &= 9768039020D-01 
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \( f_{nc}(10) \) 168.687

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( f_{nc}(10) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( clp \) and the \( csp \)

The central 100(\( p \))% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% & \quad 75\% \\
153.6 & \quad 227.7 \\
129.5 & \quad 87.5\% \\
104.5 & \quad 254.9 \\
5\% & \quad 95\% \\
89.8 & \quad 302.8 \\
2.5\% & \quad 97.5\% \\
\end{align*}
\]

Its mean is equal to 191.662 and its median is equal to 190.189

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.1411)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 286 other “replicate” pseudorandom data sets that had its Method Two logascl greater than the pre-determined \( \text{snr} \) value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12151</td>
</tr>
<tr>
<td>1</td>
<td>12018</td>
</tr>
<tr>
<td>2</td>
<td>4896</td>
</tr>
<tr>
<td>3</td>
<td>1049</td>
</tr>
<tr>
<td>4</td>
<td>158</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>
COPY LNTLDATA DATA
1 files(s) copied

C> PLNTLRA1

Given actual Type I censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \frac{\log_e(\text{fnr}) - \text{clp}}{\text{csp}} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp) based on \( n_f \):

\[
\begin{align*}
est(\text{clp}) &= .5956793964D+01 \quad \text{est(csp)} = .2925788278D+00 \\
est(\text{var(est(clp))}) &= .124842493D-01 \quad \text{est(var(est(csp))}) = .9821456650D-02 \\
est(\text{covar(est(clp),est(csp))}) &= .1081623422D-02 \quad \text{est(conceptual correlation coefficient)} = .9768030920D-01
\end{align*}
\]

<table>
<thead>
<tr>
<th>fnr</th>
<th>est(\text{y})</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.000</td>
<td>-1.1373908</td>
<td>.1276875</td>
</tr>
<tr>
<td>310.000</td>
<td>-.7526917</td>
<td>.2258176</td>
</tr>
<tr>
<td>374.000</td>
<td>-.0112116</td>
<td>.4557243</td>
</tr>
<tr>
<td>402.000</td>
<td>.1355468</td>
<td>.5539102</td>
</tr>
<tr>
<td>456.000</td>
<td>.5663392</td>
<td>.7144184</td>
</tr>
<tr>
<td>smc</td>
<td>est(y)</td>
<td>est(pf)</td>
</tr>
<tr>
<td>500.000</td>
<td>.8811784</td>
<td>.8108894</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95\% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \( \text{fnr}(10) \) 171.647

Based on the 95\(th\) Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95\% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( \text{fnr}(10) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( \text{clp} \) and the \( \text{csp} \)

The central 100(\(p\))% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>fnr</th>
<th>est(\text{y})</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>202.6</td>
<td>75%</td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
<td>223.0</td>
</tr>
<tr>
<td>100.5</td>
<td>95%</td>
<td>265.6</td>
</tr>
<tr>
<td>77.9</td>
<td>97.5%</td>
<td>333.1</td>
</tr>
<tr>
<td>64.8</td>
<td>99.9%</td>
<td>288.2</td>
</tr>
<tr>
<td>2.5%</td>
<td>.5%</td>
<td>2166</td>
</tr>
<tr>
<td>42.4</td>
<td>38</td>
<td>158.202</td>
</tr>
</tbody>
</table>

Its mean is equal to 160.636 and its median is equal to 158.202

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1891)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 703 other “replicate” pseudorandom data sets that had its Method Two losascl greater than the pre-determined smc:* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8737</td>
</tr>
<tr>
<td>1</td>
<td>12113</td>
</tr>
<tr>
<td>2</td>
<td>7237</td>
</tr>
<tr>
<td>3</td>
<td>2166</td>
</tr>
<tr>
<td>4</td>
<td>412</td>
</tr>
<tr>
<td>5</td>
<td>38</td>
</tr>
</tbody>
</table>
Given actual Type I censoring and assuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\text{loge}(\text{fnc}) - \text{clp}] / \text{csp} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp) based on \( n_{\text{total}} \)

\[
\begin{align*}
\text{est}(\text{clp}) &= .5956793964D+01 \quad \text{est}(\text{csp}) = .2814957784D+00 \\
\text{est} \{ \text{var} \{ \text{est}(\text{clp}) \} \} &= .1198487322D-01 \quad \text{est} \{ \text{var} \{ \text{est}(\text{csp}) \} \} = .9051454449D-02 \\
\text{est} \{ \text{covar} \{ \text{est}(\text{clp}), \text{est}(\text{csp}) \} \} &= .1017379384D-02 \\
\text{est(\text{conceptional correlation coefficient})} &= .9768030920D-01
\end{align*}
\]

**Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \( \text{fnc}(10) \)**

170.916

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \( \text{fnc}(10) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( \text{clp} \) and the \( \text{csp} \)

The central 100(p)% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% & \quad 75\% \\
130.1 & \quad 207.6 \\
12.5\% & \quad 87.5\% \\
105.4 & \quad 237.3 \\
5\% & \quad 95\% \\
82.5 & \quad 269.4 \\
2.5\% & \quad 97.5\% \\
68.9 & \quad 291.1 \\
.5\% & \quad 99.5\% \\
45.4 & \quad 335.6
\end{align*}
\]

Its mean is equal to 170.586 and its median is equal to 167.450

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1799)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 191 other “replicate” pseudorandom data sets that had its Method Two *loascl* greater than the pre-determined snec* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9353</td>
</tr>
<tr>
<td>1</td>
<td>12105</td>
</tr>
<tr>
<td>2</td>
<td>6845</td>
</tr>
<tr>
<td>3</td>
<td>1903</td>
</tr>
<tr>
<td>4</td>
<td>356</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
</tr>
</tbody>
</table>
Given actual Type I censoring and presuming a conceptual two-parameter log-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \left[ \log_e(\text{fnc}) - \text{clp} \right] / \text{csp} \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(\text{clp}) &= .5956793964D+01 \\
\text{est}(\text{csp}) &= .2397252434D+00 \\
\text{est}\{\text{var}(\text{est}(\text{clp}))\} &= .9987394347D-02 \\
\text{est}\{\text{var}(\text{est}(\text{csp}))\} &= .6285732256D-02 \\
\text{est}\{\text{covar}(\text{est}(\text{clp}),\text{est}(\text{csp}))\} &= .7739467195D-03 \\
\text{est}(\text{conceptual correlation coefficient}) &= .9768030920D-01
\end{align*}
\]

<table>
<thead>
<tr>
<th>func</th>
<th>est(\text{y})</th>
<th>est(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>277.00</td>
<td>-1.3661578</td>
<td>.0825445</td>
</tr>
<tr>
<td>310.00</td>
<td>-.9186420</td>
<td>.1791414</td>
</tr>
<tr>
<td>374.00</td>
<td>-.1357311</td>
<td>.4460169</td>
</tr>
<tr>
<td>402.00</td>
<td>.1654316</td>
<td>.5656978</td>
</tr>
<tr>
<td>456.00</td>
<td>.6912032</td>
<td>.7552811</td>
</tr>
<tr>
<td>snc</td>
<td>est(\text{y})</td>
<td>est(p)</td>
</tr>
<tr>
<td>500.00</td>
<td>1.0754568</td>
<td>.8589149</td>
</tr>
</tbody>
</table>

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \text{fnc}(50)

310.181

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \text{fnc}(50), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \text{clp} and the \text{csp}

The central 100(p)% of this translated pragmatic sampling distribution is bounded by the following values:

\begin{align*}
25\% & 75\% \\
295.6 & 346.5 \\
12.5\% & 87.5\% \\
278.9 & 365.3 \\
5\% & 95\% \\
262.6 & 386.4 \\
2.5\% & 97.5\% \\
252.0 & 400.2 \\
.5\% & 99.5\% \\
233.2 & 428.7
\end{align*}

Its mean is equal to 321.787 and its median is equal to 320.253

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.1411)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate
ML conceptual parameter estimates

This microcomputer program ignored 286 other “replicate” pseudorandom data sets that had its Method Two losasci greater than the pre-determined snc.* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12151</td>
</tr>
<tr>
<td>1</td>
<td>12018</td>
</tr>
<tr>
<td>2</td>
<td>4896</td>
</tr>
<tr>
<td>3</td>
<td>1049</td>
</tr>
<tr>
<td>4</td>
<td>158</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>
Given actual Type I censoring and presuming a conceptual two-parameter log-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \log_e(\text{fnc}) - \text{clp} / \text{csp} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic \( \text{est}(\text{csp}) \) based on \( n_f \)

\[
\begin{align*}
\text{est}(\text{clp}) &= .5956793964D+01 \\
\text{est}(\text{csp}) &= .2925788278D+00 \\
\text{est}([\text{var}[\text{est}(\text{clp})]]) &= .1248424293D-01 \\
\text{est}([\text{var}[\text{est}(\text{csp})]]) &= .9821456650D-02 \\
\text{est}([\text{covar}[\text{est}(\text{clp}),\text{est}(\text{csp})]]) &= .1081623422D-02 \\
\text{est}(\text{correlation coefficient}) &= .9768030920D-01 \\
\text{fnc} & \quad \text{est}(\nu) & \text{est}(\mu) \\
277.000 & -1.1373908 & .1276875 \\
310.000 & -.7526917 & .2258176 \\
374.000 & -.0112116 & .4557243 \\
402.000 & .1355468 & .5539102 \\
456.000 & .5663392 & .7144184 \\
\text{snr} & \quad \text{est}(\nu) & \text{est}(\mu) \\
500.000 & .8811784 & .8108894
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95th (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \( \text{fnc}(50) \) is 314.952

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( \text{fnc}(50) \). Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp.

The central 100\( (p) \)% of this bias-corrected pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>280.8</td>
<td>339.5</td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
</tr>
<tr>
<td>262.1</td>
<td>361.9</td>
</tr>
<tr>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>243.7</td>
<td>386.4</td>
</tr>
<tr>
<td>2.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>232.5</td>
<td>402.8</td>
</tr>
<tr>
<td>.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>210.3</td>
<td>434.6</td>
</tr>
</tbody>
</table>

Its mean is equal to 311.331 and its median is equal to 309.168

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1891)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 703 other “replicate” pseudorandom data sets that had its Method Two losascl greater than the pre-determined snr value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8737</td>
</tr>
<tr>
<td>1</td>
<td>12113</td>
</tr>
<tr>
<td>2</td>
<td>7237</td>
</tr>
<tr>
<td>3</td>
<td>2166</td>
</tr>
<tr>
<td>4</td>
<td>412</td>
</tr>
<tr>
<td>5</td>
<td>38</td>
</tr>
</tbody>
</table>
Supplemental Topic: ML-based and LR-based Lower 100(\(se\))% (One-Sided) Asymptotic Statistical Confidence Bands and Limits

Given actual Type I censoring and presuming a conceptual two-parameter log-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization:

\[ y = \frac{\text{log}(\text{fnc}) - \text{clp}}{\text{csp}} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic \(\text{est}(\text{csp})\) based on \(n_{\text{total}}\):

\[
\begin{align*}
\text{est}(\text{clp}) &= 0.5956793964D+01 \\
\text{est}(\text{csp}) &= 0.2814957784D+00 \\
\text{est} \{\text{var} [\text{est}(\text{clp})]\} &= 0.1198487322D-01 \\
\text{est} \{\text{var} [\text{est}(\text{csp})]\} &= 0.9051454449D-02 \\
\text{est} \{\text{covar} [\text{est}(\text{clp}), \text{est}(\text{csp})]\} &= 0.1017379384D-02 \\
\text{est} (\text{conceptual correlation coefficient}) &= 0.9768030920D-01
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \(\text{fnc}(50)\) = 313.492

Based on the 95\(^{th}\) Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \(\text{fnc}(50)\), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \(\text{clp}\) and \(\text{csp}\)

The central 100(\(p\))% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>(\text{fnc})</th>
<th>(\text{est}(\text{clp}))</th>
<th>(\text{est}(\text{csp}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>283.4</td>
<td>340.5</td>
</tr>
<tr>
<td>12.5%</td>
<td>265.1</td>
<td>362.1</td>
</tr>
<tr>
<td>5%</td>
<td>247.1</td>
<td>386.4</td>
</tr>
<tr>
<td>2.5%</td>
<td>236.0</td>
<td>401.9</td>
</tr>
<tr>
<td>.5%</td>
<td>213.9</td>
<td>432.8</td>
</tr>
</tbody>
</table>

Its mean is equal to 313.025 and its median is equal to 311.016

This microcomputer program ignored 2 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1799)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 591 other “replicate” pseudorandom data sets that had its Method Two \(\text{losasc}\) / greater than the pre-determined \(\text{snr}\) * value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9353</td>
</tr>
<tr>
<td>1</td>
<td>12105</td>
</tr>
<tr>
<td>2</td>
<td>6845</td>
</tr>
<tr>
<td>3</td>
<td>1903</td>
</tr>
<tr>
<td>4</td>
<td>356</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
</tr>
</tbody>
</table>
Given potential Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization
\[ y = [\log_e(\text{fnr}) - \text{clp}] / \text{csp} \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est(\text{clp})} &= 0.5927850884D+01 \quad \text{est(\text{csp})} = 0.1944865059D+00 \\
\text{est} \{ \text{var[est(\text{clp})]} \} &= 0.630416683D-02 \quad \text{est} \{ \text{var[est(\text{csp})]} \} = 0.3152083415D-02 \\
\text{est} \{ \text{cov[est(\text{clp}), est(\text{csp})]} \} &= -0.3312874752D-16 \quad \text{est(\text{correlation coefficient})} = -0.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95\% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for \text{fnr}(01)
129.764

Based on the 95\% Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95\% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \text{fnr}(01). Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(p)\% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% &= 0.110.6 \quad 75\% = 0.183.4 \\
12.5\% &= 0.087.5 \quad 95\% = 0.210.7 \\
5\% &= 0.065.8 \quad 97.5\% = 0.238.7 \\
2.5\% &= 0.052.5 \quad 99.5\% = 0.258.0 \\
0.5\% &= 0.029.4 \quad 99.9\% = 0.296.4
\end{align*}
\]

Its mean is equal to 148.376 and its median is equal to 146.099

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.0702)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 27 other “replicate” pseudorandom data sets that did not have its Method Two losascl less than the fnr* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I</th>
<th>Number of “Replicate”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Censored Datum Values</td>
<td>Pseudorandom Data Sets</td>
</tr>
<tr>
<td>0</td>
<td>19487</td>
</tr>
<tr>
<td>1</td>
<td>8721</td>
</tr>
<tr>
<td>2</td>
<td>1632</td>
</tr>
<tr>
<td>3</td>
<td>175</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>
Given potential Type I censoring and presuming a conceptual two-parameter log normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \left[ \log_{e}(fnc) - clp \right] / csp \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
\text{est}(clp) &= .5927850884D+01 \\
\text{est}(csp) &= .2283744907D+00 \\
\text{est} \{ \text{var}[\text{est}(clp)] \} &= .756000195D-02 \\
\text{est} \{ \text{var}[\text{est}(csp)] \} &= .4539000117D-02 \\
\text{est} \{ \text{covar}[\text{est}(clp),\text{est}(csp)] \} &= -.4354886956D-16 \\
\text{est}(\text{con} \text{ceptual correlation coefficient}) &= -.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for fnc(01)

129.990

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 "Replicate" Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for fnc(01), Each "Replicate" Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(\%\%) of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{array}{cc}
\text{25\%} & 75\% \\
85.4 & 159.9 \\
12.5\% & 87.5\% \\
63.6 & 189.6 \\
5\% & 95\% \\
43.2 & 220.6 \\
2.5\% & 97.5\% \\
31.9 & 241.8 \\
0.5\% & 99.5\% \\
14.6 & 284.4 \\
\end{array}
\]

Its mean is equal to 124.855 and its median is equal to 120.528

This microcomputer program ignored 0 "replicate" pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected estimate of the actual value for the probability of Type I censoring is equal to 0.1046)

This microcomputer program ignored 0 other "replicate" pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 100 other "replicate" pseudorandom data sets that did not have its Method Two lostcl less than the fnc* value selected for Type I censoring.

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of &quot;Replicate&quot; Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15577</td>
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<tr>
<td>1</td>
<td>10796</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>492</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
Given potential Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = [\log_{10}(fnc) - clp]/csp \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
est(clp) &= .5927850884D+01 \quad \text{est}(csp) = .1944865059D+00 \\
est(\text{var}(clp)) &= .6304166830D-02 \quad \text{est}(\text{var}(csp)) = .3152083415D-02 \\
est(\text{covr}(clp),\text{est}(csp)) &= .3312874752D-16 \\
est(\text{conceptional correlation coefficient}) &= -.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for fnc(10) 199.267

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for fnc(10). Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100\(\rho\)% of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% &= 25\% \\
182.0 &= 246.3 \\
12.5\% &= 87.5\% \\
160.0 &= 269.0 \\
5\% &= 95\% \\
137.9 &= 292.5 \\
2.5\% &= 97.5\% \\
123.2 &= 307.4 \\
0.5\% &= 99.5\% \\
93.4 &= 339.1
\end{align*}
\]

Its mean is equal to 218.115 and its median is equal to 218.088

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.0702)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 27 other “replicate” pseudorandom data sets that did not have its Method Two losascl less than the fnc* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19487</td>
</tr>
<tr>
<td>1</td>
<td>8721</td>
</tr>
<tr>
<td>2</td>
<td>1632</td>
</tr>
<tr>
<td>3</td>
<td>175</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>
C> COPY LNTLDATAP DATA
   1 files(s) copied
C> PLNTLRPL

Given potential Type I censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \frac{\log_e(fnc) - clp}{csp} \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
\text{est(clp)} &= .5927850884D+01 \\
\text{est(csp)} &= .2283744907D+00 \\
\text{est} \{\text{var(est(clp))}\} &= .756000195D-02 \\
\text{est} \{\text{var(est(csp))}\} &= .4539000117D-02 \\
\text{est} \{\text{covar(est(clp),est(csp))}\} &= -.4354886956D-16 \\
\text{est(conceptual correlation coefficient)} &= -.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Tolerance Limit that Allegedly Bound the Actual Value for fnc(10)

200.798

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for fnc(10), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(p)th % of this translated pragmatic sampling distribution is bounded by the following values:

\[
\begin{align*}
25\% & \quad 118.5 \\
75\% & \quad 228.2 \\
12.5\% & \quad 87.5\% \\
153.5 & \quad 253.7 \\
5\% & \quad 95\% \\
111.8 & \quad 280.1 \\
2.5\% & \quad 97.5\% \\
96.9 & \quad 298.6 \\
0.5\% & \quad 99.5\% \\
69.7 & \quad 334.1
\end{align*}
\]

Its mean is equal to 194.076 and its median is equal to 192.587

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1046)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 21 other “replicate” pseudorandom data sets that did not have its Method Two losasc1 less than the fnc* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15675</td>
</tr>
<tr>
<td>1</td>
<td>10743</td>
</tr>
<tr>
<td>2</td>
<td>3127</td>
</tr>
<tr>
<td>3</td>
<td>442</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>
Given potential Type I censoring and presuming a conceptual two-parameter lognormal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \log(nlc) - clp/csp \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic est(csp)

\[
\begin{align*}
est(clp) &= 0.5927850884D+01 \quad \text{est}(csp) = 0.1944865059D+00 \\
est(\text{var}(clp)) &= 0.6304166830D-02 \quad \text{est}(\text{var}(csp)) = 0.3152083415D-02 \\
est(\text{covar}(clp,csp)) &= -0.3312874752D-16 \\
est(\text{conceplional correlation coefficient}) &= -0.7431770971D-14
\end{align*}
\]

**Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit**

that Allegedly Bound the Actual Value for \( fnl(50) \)

313.397

Based on the 95th Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( fnl(50) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence region that Allegedly Includes the Actual Values for the \( clp \) and the \( csp \)

The central 100(p)\% of this translated pragmatic sampling distribution is bounded by the following values:

25% 300.9 342.9
12.5% 87.5% 286.8 358.5
5% 95% 273.0 375.3
2.5% 97.5% 264.1 386.8
0.5% 99.5% 247.2 410.0

Its mean is equal to 322.467 and its median is equal to 321.555

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The ML estimate of the actual value for the probability of Type I censoring is equal to 0.0702)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 27 other “replicate” pseudorandom data sets that had its Method Two losascl greater than the pre-determined \( \text{smc*} \) value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19487</td>
</tr>
<tr>
<td>1</td>
<td>8721</td>
</tr>
<tr>
<td>2</td>
<td>1632</td>
</tr>
<tr>
<td>3</td>
<td>175</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>
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Given potential Type I censoring and presuming a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization
\[ y = \log(e) (fn) - \text{clp}/csp \]

and with extrapolated Version LS statistical bias-correction factors and an extrapolated exact multiplicative median statistical bias-correction factor for generic \( \text{est}(csp) \)

\[
\begin{align*}
\text{est}(\text{clp}) = 0.5927850884D+01 & \quad \text{est}(\text{csp}) = 0.2283744907D+00 \\
\text{est}\{\text{var}(\text{est}(\text{clp}))\} = 0.756000195D-02 & \quad \text{est}\{\text{var}(\text{est}(\text{csp}))\} = 0.4539000117D-02 \\
\text{est}\{\text{covar}(\text{est}(\text{clp}),\text{est}(\text{csp}))\} = -0.4354886956D-16 & \quad \text{est}(\text{conceptual correlation coefficient}) = -0.7431770971D-14
\end{align*}
\]

Pragmatic Bias-Corrected Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bound the Actual Value for \( fn(50) \)

315.602

Based on the 95\textsuperscript{th} Percentile of the Pragmatic Sampling Distribution Comprised of 30000 “Replicate” Realization Values for the Likelihood Ratio Lower 95% (One-Sided) Asymptotic Statistical Confidence Limit that Allegedly Bounds the Actual Value for \( fn(50) \), Each “Replicate” Realization Value Computed Using 360 Points on the Boundary of the LR-Based Joint Asymptotic Statistical Confidence Region that Allegedly Includes the Actual Values for the clp and the csp

The central 100(\( p \))% of this translated pragmatic sampling distribution is bounded by the following values:

<table>
<thead>
<tr>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>290.4</td>
<td>388.1</td>
</tr>
<tr>
<td>12.5%</td>
<td>87.5%</td>
</tr>
<tr>
<td>274.6</td>
<td>355.9</td>
</tr>
<tr>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>259.1</td>
<td>375.3</td>
</tr>
<tr>
<td>2.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>249.3</td>
<td>388.8</td>
</tr>
<tr>
<td>0.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>231.2</td>
<td>416.3</td>
</tr>
</tbody>
</table>

Its mean is equal to 314.877 and its median is equal to 313.522

This microcomputer program ignored 0 “replicate” pseudorandom data sets that had all of its datum values Type I censored. (The median bias-corrected ML estimate of the actual value for the probability of Type I censoring is equal to 0.1046)

This microcomputer program ignored 0 other “replicate” pseudorandom data sets that did not produce accurate ML conceptual parameter estimates

This microcomputer program ignored 100 other “replicate” pseudorandom data sets that had its Method Two losascl greater than the pre-determined snc\* value selected for Type I censoring

<table>
<thead>
<tr>
<th>Number of Type I Censored Datum Values</th>
<th>Number of “Replicate” Pseudorandom Data Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15577</td>
</tr>
<tr>
<td>1</td>
<td>10796</td>
</tr>
<tr>
<td>2</td>
<td>3179</td>
</tr>
<tr>
<td>3</td>
<td>492</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
(b) **Strength (Resistance) Data:** Recall that ML analyses for strength (resistance) data traditionally employ only the parameterization \( y = clp + csp \cdot s \). Accordingly, microcomputer programs \( N2B, L2B, SEV2B, \) and \( LEV2B \) respectively employ this parameterization for conceptual (two-parameter) normal, logistic, smallest-extreme-value, and largest-extreme-value strength (resistance) distribution models and compute ML-based and LR-based lower 100(\( scp \))% (one-sided) asymptotic statistical confidence bands that allegedly bound the actual CDF.

**ML-Based and LR-Based Lower 100(\( scp \))% (One-Sided) Asymptotic Statistical Confidence Limits for Both Life (Endurance) and Strength (Resistance) Data:** ML-based and LR-based lower 100(\( scp \))% (one-sided) asymptotic statistical confidence limits that allegedly bound the presumed conceptual two-parameter distribution metric value given any \( y(pf) \) value of specific interest can also be computed using the numerical computation procedure previously used to establish the ML-based and LR-based asymptotic joint confidence regions that allegedly contain the actual values for the \( clp \) and the \( csp \). However, these limits are established by a \( scp \)-based value of Pearson’s central \( \chi^2 \) conceptual sampling distribution with (only) one statistical degree of freedom — because, given the \( scp \) value of specific interest, \( y(\text{scp}) \) establishes a unique path to the respective boundaries of the ML-based and LR-based joint asymptotic confidence regions.

(a) **Life (Endurance) Data:** The \( LSEV(J)CAC \) and \( LN(J)CACN \) series of microcomputer programs compute ML-based and LR-based lower 100(\( scp \))% (one-sided) asymptotic statistical confidence limits. As indicated in Table 8F.3, the ML-based statistical confidence limits for the linear parameterizations (2) and (3) are identical to those computed by the \( LSEV(J)AAC \) and \( LN(J)AACN \) series of microcomputer programs.

<table>
<thead>
<tr>
<th>CDF Parameterization</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptual (Two-Parameter)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weibull Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using Method One</td>
<td>34.584</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using Method Two</td>
<td></td>
<td>86.868</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using the Joint Asymptotic Confidence Region</td>
<td>36.167</td>
<td><strong>34.585</strong></td>
<td>86.868</td>
<td>9.598</td>
</tr>
<tr>
<td>Computed Using the Likelihood Ratio Method</td>
<td>58.511</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conceptual (Two-Parameter)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log_( e ) Normal Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using Method One</td>
<td>158.933</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using Method Two</td>
<td>112.714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computed Using the Joint Asymptotic Confidence Region</td>
<td>114.499</td>
<td><strong>112.735</strong></td>
<td>158.933</td>
<td>28.526</td>
</tr>
<tr>
<td>Computed Using the Likelihood Ratio Method</td>
<td>128.175</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The 34.585 value should be exactly equal to 34.584 and the 112.735 value should be exactly equal to 112.714. However, the est(\text{conceptual correlation coefficient}) for this parameterization is so large that the elliptical joint asymptotic confidence region is very long and slender. Thus, given equal spacing of the 3600 boundary points in terms of their angular orientation, too few of these points lie at the extreme ends of the joint confidence region to generate exact agreement with the corresponding propagation of variability estimate.
Presuming a conceptual (two-parameter) normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(pf) = clp + csp \cdot s(pf) \]

(and with no statistical bias corrections)

\[
\begin{align*}
est(clp) &= -7438137674D+01, \quad est(csp) = .8926409259D-01 \\
est[var(est(clp))] &= .431538215D+01, \quad est[var(est(csp))] = .5899274299D-03 \\
est[covar(est(clp),est(csp))] &= -.5012866877D-01, \quad est(conceptual correlation coefficient) = -.9935265033D+00
\end{align*}
\]

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

\[
\begin{align*}
x &\quad est(y) &\quad est(pf) \\
100.000 &\quad 1.4882716 &\quad .9316604 \\
95.000 &\quad 1.0419511 &\quad .8512828 \\
90.000 &\quad .5956307 &\quad .7242890 \\
85.000 &\quad .1493102 &\quad .5593456 \\
80.000 &\quad -.2970103 &\quad .3832293 \\
70.000 &\quad -1.1896512 &\quad .1170918 \\
65.000 &\quad -1.6359717 &\quad .0509228
\end{align*}
\]

at \( s = 100.000 \)

Lower Boundary Point =

\[
\begin{align*}
est(s) &= 17.407 \\
est(s) &= 57.226
\end{align*}
\]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point =

\[
\begin{align*}
x &\quad est(s) \\
92.302 &\quad 100.000 \\
88.560 &\quad 95.000 \\
84.262 &\quad 90.000 \\
78.768 &\quad 85.000 \\
71.308 &\quad 80.000 \\
52.421 &\quad 75.000 \\
42.275 &\quad 65.000
\end{align*}
\]

at \( s(01) = 100.000 \)

Lower Boundary Point =

\[
\begin{align*}
est(s) &= 26.305 \\
est(s) &= 57.226
\end{align*}
\]
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C> L2B

Presuming a conceptual (two-parameter) logistic distribution strength (resistance) model in ML analysis with the conventional parameterization

\( y(pf) = clp + csp \times pf \)

(and with no statistical bias corrections)

\( \text{est}(clp) = -.1316853685D+02 \quad \text{est}(csp) = .1572425344D-00 \)
\( \text{est}\{\text{var}(\text{est}(clp))\} = .1719712841D+02 \quad \text{est}\{\text{var}(\text{est}(csp))\} = .2331274510D-02 \)
\( \text{est}\{\text{covar}(\text{est}(clp),\text{est}(csp))\} = -.19923002537D+00 \quad \text{est}(\text{conceptual correlation coefficient}) = -.9950172579D+00 \)

<table>
<thead>
<tr>
<th>( s )</th>
<th>( \text{est}(y) )</th>
<th>( \text{est}(pf) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.000</td>
<td>2.5557166</td>
<td>.9279566</td>
</tr>
<tr>
<td>95.000</td>
<td>1.7965039</td>
<td>.8543960</td>
</tr>
<tr>
<td>90.000</td>
<td>.9832912</td>
<td>.7277608</td>
</tr>
<tr>
<td>85.000</td>
<td>.1970786</td>
<td>.5491108</td>
</tr>
<tr>
<td>80.000</td>
<td>-.5891341</td>
<td>.356336</td>
</tr>
<tr>
<td>70.000</td>
<td>-.21615594</td>
<td>.1032560</td>
</tr>
<tr>
<td>65.000</td>
<td>-.29477721</td>
<td>.0498419</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 2 Statistical Degrees of Freedom)

**Lower Boundary Point**

\( \text{at } s = \)

92.637 100.000
88.915 95.000
84.306 90.000
77.126 85.000
65.976 80.000
38.630 75.000
24.317 65.000

\( \text{at } s(01) \)

**Lower Boundary Point**

\( \text{est}(s) = \)

\(-6.021\) 54.524

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

**Lower Boundary Point**

\( \text{at } s = \)

91.941 100.000
88.420 95.000
84.208 90.000
78.558 85.000
70.519 80.000
50.101 75.000
39.234 65.000

\( \text{at } s(01) \)

**Lower Boundary Point**

\( \text{est}(s) = \)

16.101 54.524
Supplemental Topic: ML-based and LR-based Lower 100(scp)% (One-Sided) Asymptotic Statistical Confidence Bands and Limits

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C> SEV2B

Presuming a conceptual (two-parameter) smallest-extreme-value normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(pf) = clp + csp \cdot s(pf) \]

(and with no statistical bias corrections)

\[ \text{est}(clp) = -1.003518158D+02 \quad \text{est}(csp) = .1137870314D+00 \]
\[ \text{est} \{ \text{var} [\text{est}(clp)] \} = .8451907194D+01 \quad \text{est} \{ \text{var} [\text{est}(csp)] \} = .1057426055D-02 \]
\[ \text{est} \{ \text{cov} [\text{est}(clp), \text{est}(csp)] \} = -.9417710941D-01 \quad \text{est} (\text{conceptual correlation coefficient}) = -.9961918698D+00 \]

<table>
<thead>
<tr>
<th>( s )</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.00</td>
<td>1.3435216</td>
<td>.9783449</td>
</tr>
<tr>
<td>95.00</td>
<td>.7745864</td>
<td>8.857875</td>
</tr>
<tr>
<td>90.00</td>
<td>.2056512</td>
<td>.7072173</td>
</tr>
<tr>
<td>85.00</td>
<td>-.3632839</td>
<td>.5011196</td>
</tr>
<tr>
<td>80.00</td>
<td>-.9322191</td>
<td>.3254295</td>
</tr>
<tr>
<td>70.00</td>
<td>-.20700894</td>
<td>.1185390</td>
</tr>
<tr>
<td>65.00</td>
<td>-.26390245</td>
<td>.0689394</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom)

Lower Boundary Point =

\[ s = 94.256 \quad \text{at s} = 100.000 \]
\[ 90.171 \quad 95.000 \]
\[ 84.443 \quad 90.000 \]
\[ 75.300 \quad 85.000 \]
\[ 63.767 \quad 80.000 \]
\[ 38.817 \quad 75.000 \]
\[ 26.079 \quad 65.000 \]
\[ \text{at s(01)} \]

Lower Boundary Point =

\[ \text{est}(s) = -18.174 \quad 47.765 \]

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

Lower Boundary Point =

\[ s = 94.122 \quad \text{at s} = 100.000 \]
\[ 90.254 \quad 95.000 \]
\[ 85.168 \quad 90.000 \]
\[ 77.969 \quad 85.000 \]
\[ 68.828 \quad 80.000 \]
\[ 48.310 \quad 75.000 \]
\[ 37.705 \quad 65.000 \]
\[ \text{at s(01)} \]

Lower Boundary Point =

\[ \text{est}(s) = 0.707 \quad 47.765 \]
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C> LEV2B

Presuming a conceptual (two-parameter) largest-extreme-value normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(pf) = clp + csp \cdot s(pf) \]

(and with no statistical bias corrections)

\[
\begin{align*}
est(clp) &= -6761135294D+01 \quad \text{est}(csp) = .8725876010D-01 \\
est(\text{var}(clp)) &= .3662399509D+01 \quad \text{est}(\text{var}(csp)) = .5512958654D-03 \\
est(\text{covar}(clp,csp)) &= -.4531484122D+01 \quad \text{est( conceptual correlation coefficient)} = -.9906437067D+00
\end{align*}
\]

<table>
<thead>
<tr>
<th>s</th>
<th>est(y)</th>
<th>est(pf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.000</td>
<td>1.9647407</td>
<td>.8691911</td>
</tr>
<tr>
<td>95.000</td>
<td>1.5284469</td>
<td>.8050328</td>
</tr>
<tr>
<td>90.000</td>
<td>.1.0921531</td>
<td>.7149853</td>
</tr>
<tr>
<td>85.000</td>
<td>.6558593</td>
<td>.5951178</td>
</tr>
<tr>
<td>80.000</td>
<td>.2195655</td>
<td>.4480423</td>
</tr>
<tr>
<td>70.000</td>
<td>-.6530221</td>
<td>.1464109</td>
</tr>
<tr>
<td>65.000</td>
<td>-.1.0893159</td>
<td>.0511885</td>
</tr>
</tbody>
</table>

Points on the Lower Band of the 90% (Two-Sided) Asymptotic Statistical Confidence Bands that Allegedly Bound the Actual CDF

Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 2 Statistical Degrees of Freedom)

<table>
<thead>
<tr>
<th>Lower Boundary Point =</th>
<th>at s =</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.718</td>
<td>100.000</td>
</tr>
<tr>
<td>87.111</td>
<td>95.000</td>
</tr>
<tr>
<td>83.186</td>
<td>90.000</td>
</tr>
<tr>
<td>78.562</td>
<td>85.000</td>
</tr>
<tr>
<td>72.397</td>
<td>80.000</td>
</tr>
<tr>
<td>53.545</td>
<td>75.000</td>
</tr>
<tr>
<td>42.328</td>
<td>65.000</td>
</tr>
</tbody>
</table>

at s(01)

<table>
<thead>
<tr>
<th>Lower Boundary Point =</th>
<th>est(s) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.678</td>
<td>59.982</td>
</tr>
</tbody>
</table>

Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

<table>
<thead>
<tr>
<th>Lower Boundary Point =</th>
<th>at s =</th>
</tr>
</thead>
<tbody>
<tr>
<td>89.751</td>
<td>100.000</td>
</tr>
<tr>
<td>86.229</td>
<td>95.000</td>
</tr>
<tr>
<td>82.488</td>
<td>90.000</td>
</tr>
<tr>
<td>78.227</td>
<td>85.000</td>
</tr>
<tr>
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at s(01)

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Supplemental Topic: ML-based and LR-based Lower 100(scp)% (One-Sided) Asymptotic Statistical Confidence Bands and Limits

C> LSEV1CAC

Presuming Type I censoring and a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = csp \cdot \log_e(fnc) - clp \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= .6061362736D+01 \quad \text{est}(csp) = .4731943406D+01 \\
\text{est}\{\text{var}[\text{est}(clp)]\} &= .9059101593D-02 \quad \text{est}\{\text{var}[\text{est}(csp)]\} = .3013638227D+01 \\
\text{est}\{\text{covar}[\text{est}(clp),\text{est}(csp)]\} &= .1956788216D-01 \quad \text{est}(\text{conceptual correlation coefficient}) = .1184283562D+00
\end{align*}
\]

\[
\begin{array}{ccc}
\text{fnc} & \text{est}(y) & \text{est}(pf) \\
277.00 & -2.0694929 & .1186053 \\
310.00 & -1.5368900 & .1934980 \\
374.00 & -.6487823 & .4070717 \\
402.00 & -.3071535 & .5207523 \\
456.00 & .2892640 & .7369587 \\
500.00 & .7251484 & .8731865
\end{array}
\]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for fnc(01)

36.167 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

58.511 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

C> LSEV2CAC

Presuming Type I censoring and a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest extreme-value distribution in ML analysis with the parameterization

\[ y = clp + csp \cdot \log_e(fnc) \]

and with no statistical bias corrections

\[
\begin{align*}
\text{est}(clp) &= -.2868202543D+02 \quad \text{est}(csp) = .4731943406D+01 \\
\text{est}\{\text{var}[\text{est}(clp)]\} &= .1120467627D+03 \quad \text{est}\{\text{var}[\text{est}(csp)]\} = .3013638227D+01 \\
\text{est}\{\text{covar}[\text{est}(clp),\text{est}(csp)]\} &= -.1835934856D+02 \quad \text{est}(\text{conceptual correlation coefficient}) = -.9991071169D+00
\end{align*}
\]

\[
\begin{array}{ccc}
\text{fnc} & \text{est}(y) & \text{est}(pf) \\
277.00 & -2.0694929 & .1186053 \\
310.00 & -1.5368900 & .1934980 \\
374.00 & -.6487823 & .4070717 \\
402.00 & -.3071535 & .5207523 \\
456.00 & .2892640 & .7369587 \\
500.00 & .7251484 & .8731865
\end{array}
\]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for fnc(01)

34.585 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

58.511 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
C> LSEV3CAC

Presuming Type I censoring and a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = \left[ \log_e(fnc) - \text{clp} \right]/\text{csp} \]

and with no statistical bias corrections

\[
\begin{array}{ccc}
\text{est(clp)} & \text{est(csp)} & \text{est(\text{var[est(clp)]})} = .9059101593D-02 & \text{est(\text{var[est(csp)]})} = .6010809287D-02 \\
\text{est(\text{covar[est(clp),est(csp)]})} = -.8739060349D-03 & \text{est(\text{conceptsual correlation coefficient})} = -.1184283562D+00 \\
\end{array}
\]

\[ \begin{array}{ccc}
\text{fnc} & \text{est(y)} & \text{est(\text{pf})} \\
277.000 & -2.0694929 & .1186053 \\
310.000 & -1.5368900 & .1934980 \\
374.000 & -.6487823 & .4070717 \\
402.000 & -.3071535 & .5207523 \\
456.000 & .2892640 & .7369587 \\
\end{array} \]

\[ \begin{array}{cc}
\text{snc} & \text{est(y)} & \text{est(\text{pf})} \\
500.000 & .7251484 & .8731865 \\
\end{array} \]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits
that Allegedly Bound the Actual Value for \text{fnc(01)}

86.868 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

58.511 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

---

C> LSEV4CAC

Presuming Type I censoring and a conceptual two-parameter Weibull distribution fatigue life model, but employing the conceptual (two-parameter) smallest-extreme-value distribution in ML analysis with the parameterization

\[ y = \left[ \log_e(fnc)/\text{csp} \right] - \text{clp} \]

and with no statistical bias corrections

\[
\begin{array}{ccc}
\text{est(clp)} & \text{est(csp)} & \text{est(\text{var[est(clp)]})} = .1120467672D+03 & \text{est(\text{var[est(csp)]})} = .6010809287D-02 \\
\text{est(\text{covar[est(clp),est(csp)]})} = -.8199326557D+00 & \text{est(\text{conceptsual correlation coefficient})} = -.9991071169D+00 \\
\end{array}
\]

\[ \begin{array}{ccc}
\text{fnc} & \text{est(y)} & \text{est(\text{pf})} \\
277.000 & -2.0694929 & .1186053 \\
310.000 & -1.5368900 & .1934980 \\
374.000 & -.6487823 & .4070717 \\
402.000 & -.3071535 & .5207523 \\
456.000 & .2892640 & .7369587 \\
\end{array} \]

\[ \begin{array}{cc}
\text{snc} & \text{est(y)} & \text{est(\text{pf})} \\
500.000 & .7251484 & .8731865 \\
\end{array} \]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits
that Allegedly Bound the Actual Value for \text{fnc(01)}

9.598 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

58.511 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
Presuming Type I censoring and a conceptual two-parameter log-$e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = csp \cdot \log_e(fnc) - clp \]

and with no statistical bias corrections

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<td>374.000</td>
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<td>456.000</td>
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<td>500.000</td>
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</table>

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \( fnc(01) \)

114.499 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 1 Statistical Degree of Freedom)

128.175 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

---

Presuming Type I censoring and a conceptual two-parameter log-$e$-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = clp + csp \cdot \log_e(fnc) \]

and with no statistical bias corrections

<table>
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<tr>
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<th>( \text{est}(y) )</th>
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<tbody>
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<td>0.8589149</td>
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</tbody>
</table>

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \( fnc(01) \)

112.735 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 1 Statistical Degree of Freedom)

128.175 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
C> LN3CACN

Presuming Type I censoring and a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \log(e) \cdot (\text{fnc}) - \text{clp}/\text{csp} \]

and with no statistical bias corrections

\[
\begin{array}{ccc}
\text{est(clp)} &=& \text{.5959062671D+01} \\
\text{est(csp)} &=& \text{.2412998143D+00} \\
\text{est(var[est(clp)]}) &=& \text{.9921960558D-02} \\
\text{est(var[est(csp)])} &=& \text{.6305038312D-02} \\
\text{est(covar[est(clp),est(csp)])} &=& \text{.6974504622D-03} \\
\text{est(conceptual correlation coefficient)} &=& \text{.8818013378D-01} \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{finc} & \text{est(y)} & \text{est(pf)} \\
277.000 & -1.3881578 & .0825445 \\
310.000 & -.9186420 & .1791414 \\
374.000 & -.1357311 & .4460169 \\
402.000 & .1654316 & .5656978 \\
456.000 & .6912032 & .7552811 \\
\text{src} & \text{est(y)} & \text{est(pf)} \\
500.000 & 1.0754568 & .8589149 \\
\end{array}
\]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \text{finc(01)}

158.933 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \text{clp} and the \text{csp} (with 1 Statistical Degree of Freedom)

128.175 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

C> LN4CACN

Presuming Type I censoring and a conceptual two-parameter loge-normal distribution fatigue life model, but employing the conceptual (two-parameter) normal distribution in ML analysis with the parameterization

\[ y = \{\log(e)(\text{fnc})\}/\text{clp} - \text{clp} \]

and with no statistical bias corrections

\[
\begin{array}{ccc}
\text{est(clp)} &=& \text{-.2484842180D+02} \\
\text{est(csp)} &=& \text{.2397252434D+00} \\
\text{est(var[est(clp)]}) &=& \text{.6703904559D+02} \\
\text{est(var[est(csp)])} &=& \text{.6285732256D-02} \\
\text{est(covar[est(clp),est(csp)])} &=& \text{-.6483112811D+02} \\
\text{est(conceptual correlation coefficient)} &=& \text{-9987153578D+00} \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{finc} & \text{est(y)} & \text{est(pf)} \\
277.000 & -1.3881578 & .0825445 \\
310.000 & -.9186420 & .1791414 \\
374.000 & -.1357311 & .4460169 \\
402.000 & .1654316 & .5656978 \\
456.000 & .6912032 & .7552811 \\
\text{src} & \text{est(y)} & \text{est(pf)} \\
500.000 & 1.0754568 & .8589149 \\
\end{array}
\]

Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \text{finc(01)}

28.526 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \text{clp} and the \text{csp} (with 1 Statistical Degree of Freedom)

128.175 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
(h) **Strength (Resistance) Data:** Microcomputer programs N2C, I2C, SEV2C, and LEV2C compute ML-based and LR lower 100(scp)% (one-sided) asymptotic statistical confidence limits that allegedly bound the metric pertaining to the \(p^{th}\) percentile of the presumed conceptual (two-parameter) strength (resistance) distribution, where scp and \(p\) are input values that are specified in microcomputer file ASDDATA. The respective ML-based lower 100(scp)% (one-sided) asymptotic statistical confidence limits are identical to those computed using Method Two in microcomputer programs N2ALCL, L2ALCL, SEV2ALCL, and LEV2ALCL. Note that the respective lower (one-sided) asymptotic statistical confidence limits differ very markedly — and that even the symmetrical conceptual (two-parameter) normal and logistic distributions differ markedly. Clearly it is impractical to attempt to discern between any two plausible strength (resistance) distributions due the extremely large number of datum values (nominally identical specimens) that would be required to be reasonably confident that the associated statistical test of hypothesis will have a practical value for its statistical power. Thus, it is not statistically rational to attempt to compute \(A\)-basis or even \(B\)-basis statistical tolerance limits for using (quantal response) fatigue strength data.

**Summary:** Given log\(_e\)-normal life (endurance) datum values with no censoring pragmatic bias-corrected LR-based lower 100(scp)% (one-sided) statistical confidence and tolerance limits are equivalent, for practical purposes, to correspondence pragmatic bias-corrected Method Two lower 100(scp)% (one-sided) statistical confidence and tolerance limits. In particular, both sets of lower 100(scp)% (one-sided) statistical confidence and tolerance limits are equivalent, for practical purposes, to exact (unbiased) lower 100(scp)% (one-sided) statistical confidence and tolerance limits when the actual value for the coefficient of variation is less than about 0.2. This equivalence provides an intuitive rationale for extending these analyses to log\(_e\)-normal life (endurance) datum value with Type I censoring.

```
C> COPY ASDDATA DATA
  1 files(s) copied
C> N2C

Presuming a conceptual (two-parameter) normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(p) = clp + csp \cdot s(p) \]

(and with no statistical bias corrections)

\[
\begin{align*}
est(clp) &= -7438137674D+01 & \text{est(csp)} &= .8926409259D-01 \\
est[\text{var(est(clp))}] &= .4315338215D+01 & \text{est[\text{var(est(csp))}] } &= .5899274299D-03 \\
est[\text{covar(est(clp),est(csp))}] &= -.5012866877D-01 & \text{est(conceptual correlation coefficient)} &= -.9935265033D+00 \\
\end{align*}
\]

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Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for s(01)

34.074 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

37.678 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
```
Presuming a conceptual (two-parameter) logistic distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(p_f) = clp + csp \cdot s(p_f) \]

(and with no statistical bias corrections)

\[
\begin{align*}
    \text{est}(clp) &= -.1316853685D+02 & \text{est}(csp) &= .1572425344D-00 \\
    \text{est}([\text{var}(\text{est}(clp))] &= .1719712841D+02 & \text{est}([\text{var}(\text{est}(csp))] &= .2331274510D-02 \\
    \text{est}([\text{covar}(\text{est}(clp),\text{est}(csp))] &= -.19923002537D+00 & \text{est}(\text{conceptual correlation coefficient}) &= -.9950172579D+00
\end{align*}
\]

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<th>est(p_f)</th>
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Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \( s(0.1) \)

22.363 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 1 Statistical Degree of Freedom)

30.027 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method

---

Presuming a conceptual (two-parameter) smallest-extreme-value normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(p_f) = clp + csp \cdot s(p_f) \]

(and with no statistical bias corrections)

\[
\begin{align*}
    \text{est}(clp) &= -.1003518158D+02 & \text{est}(csp) &= .1137870314D+00 \\
    \text{est}([\text{var}(\text{est}(clp))] &= .8451907194D+01 & \text{est}([\text{var}(\text{est}(csp))] &= .1057426055D-02 \\
    \text{est}([\text{covar}(\text{est}(clp),\text{est}(csp))] &= -.9417710941D-01 & \text{est}(\text{conceptual correlation coefficient}) &= -.9961918698D+00
\end{align*}
\]

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Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits that Allegedly Bound the Actual Value for \( s(0.1) \)

10.789 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the \( clp \) and the \( csp \) (with 1 Statistical Degree of Freedom)

17.593 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method
Presuming a conceptual (two-parameter) largest-extreme-value normal distribution strength (resistance) model in ML analysis with the conventional parameterization

\[ y(pf) = clp + csp \cdot s(pf) \]

(and with no statistical bias corrections)

\[
\begin{align*}
\text{est}(clp) &= -6761135294D+01 \\
\text{est}(csp) &= .8725876010D+01 \\
\text{est}[\text{var}(clp)] &= .3662399509D+01 \\
\text{est}[\text{var}(csp)] &= .5512958654D+03 \\
\text{est}[\text{covar}(clp, csp)] &= -.4531384182D-01 \\
\text{est}(\text{conceptual correlation coefficient}) &= -.9906437076D+00
\end{align*}
\]

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Lower 95% (One-Sided) Asymptotic Statistical Confidence Limits
that Allegedly Bound the Actual Value for s(01)

42.814 - Computed Using 3600 Points on the Boundary of the Elliptical Joint Asymptotic Statistical Confidence Region for the Actual Values of the clp and the csp (with 1 Statistical Degree of Freedom)

45.238 - Computed Using 3600 Points on the Boundary of the Corresponding Region Established by the Likelihood Ratio Method